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RESEARCH ARTICLE

A Heat Transfer Problem with a Velocity Slip in Porous Media: A Galerkin Approach

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ABSTRACT

In this paper a steady flow of a viscous fluid of finite depth in a porous medium over a fixed horizontal, impermeable bottom with a velocity slip is studied. A Galerkin Method of solution is considered to solve Momentum and Energy equations as the truncation errors both in general and special cases may result in complex situations to get the analytical form of solutions. Using the obtained velocity profile, the Temperature, Mean velocity, Mean Temperature, and heat transfer rates (Nusselt Number) on the free surface as well as on the bottom plate are obtained. The effect of velocity slip in the obtained fields is studied wherever possible. A special case of low porosity is studied and the results are illustrated graphically.

Keywords: Porosity coefficient, Porous Medium, Temperature, Velocity.

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1. Introduction

Bevers *et al.* [1] considered in addition to the classical Darcy's law, a slip boundary condition at the interface between the porous medium and a clear medium. Subsequently, Saffman [2] provided a theoretical justification for the Bevers *et al.* [1] boundary condition. Taylor [3], and Richardson [4] carried out the exact analysis of the situation assuming a different model together with Bevers's *et al.* [1] interface condition. Forced convective heat flow of a viscous liquid of finite depth in a porous medium over a fixed horizontal impermeable plate was studied by Moinuddin *et al.* [5] in the year 2011. In the year 2018 Khaja Moinuddin [6] studied fluid flow and heat transfer of a viscous fluid of finite depth over a fixed thermally insulated bottom. In 2013 Taamneh *et al.* [7] studied slip flow and heat transfer in a saturated porous micro-channel. Shah *et al.* [8] in the year 2021 studied the effect of slip on a mixed convective flow and heat transfer of fluid through porous medium.

The flow is generated by a constant horizontal pressure gradient parallel to the fixed bottom. The momentum equation considered is the generalized darcy's law proposed by Moto *et al.* [9] which takes into account the convective acceleration and the Newtonian viscous stresses in addition to the classical Darcy force.

The Momentum and Energy equations are solved using Galerkin Method in order to avoid the difficulties of getting solution because of truncation error both in general and a special case of porosity. Using this technique the velocity profile is obtained first and with the help of this the other fields such as mean velocity, temperature, mean temperature and nusselt numbers on the boundaries are calculated and their variations are illustrated graphically. A special case of Low porosity coefficient (large α) is also discussed.

2. Mathematical Formulation

Consider the steady forced convective flow of a viscous liquid through a porous medium of viscosity coefficient μ and of finite depth H over a fixed horizontal impermeable bottom with a velocity slip. The flow is generated by a constant pressure gradient parallel to the plate. The free surface is exposed to the atmosphere kept at temperature T_1.

With reference to a rectangular Cartesian coordinate system with the origin O on the bottom, the X-axis in the flow direction (i.e., parallel to the applied pressure gradient) and the Y-axis vertically

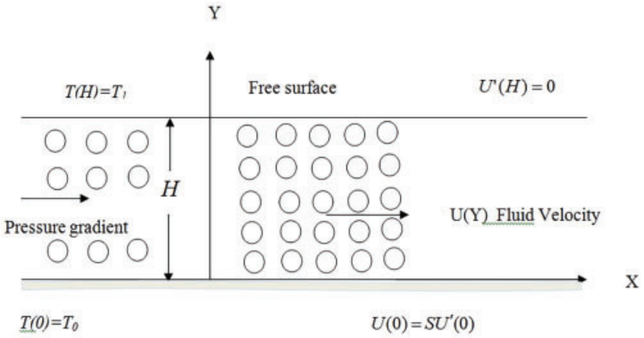

Fig. 1. Flow configuration.

upwards. The bottom is represented as Y = 0 and the free surface as Y = H. Let the flow be characterized by a velocity U = (U(Y), 0, 0). This choice of velocity evidently satisfies the continuity equation $\nabla.U = 0$. This is exhaustively mentioned in the flow configuration given in Fig. 1.

2.1. *Basic Equations*

Let the convective flow be characterized by the velocity field U = (U(Y), 0, 0) and the temperature T (Y). The choice of the velocity satisfies the continuity equation:

$$divU = 0 \tag{1}$$

The momentum equation:

$$-\frac{\partial P}{\partial X} + \mu\frac{d^2U}{dY^2} - \frac{\mu U}{k^*} = 0 \tag{2}$$

The energy equation:

$$\rho c U\frac{dT}{dX} = K\frac{d^2T}{dY^2} + \mu\left(\frac{dU}{dY}\right)^2 \tag{3}$$

2.2. *Boundary Conditions*

Since the fluid slips on the bottom,

$$\mathrm{U}(0) = S\frac{dU}{dY}\mid_{Y=0} \tag{4}$$

where S is the slip parameter.

$$\text{At the free surface shear stress } = \mu\frac{dU}{dY} = 0 \text{ on Y = H.} \tag{5}$$

Also,

$$\mathrm{T}\ (0) = \mathrm{T}_0 \tag{6}$$

and

$$\mathrm{T}\ (\mathrm{H}) = \mathrm{T}_1 \tag{7}$$

where T_0 is the bottom temperature and T_1 is the atmosphere temperature.

In terms of the non-dimensional variables defined hereunder:

$$\mathrm{Y = ay;\ X = ax;\ H = ah;}\ U = \frac{\mu u}{\rho a^2};\ P = \frac{\mu^2 p}{\rho a^2};\ T = T_0 + (T_1 - T_0)\,\theta;\ \mathrm{Pr} = \frac{\mu c}{k};\ k^* = \frac{a^2}{\alpha^2};$$

$$E = \frac{\mu^3}{\rho^2 a^2 K\,(T_1 - T_0)};\ -\frac{\partial P}{\partial X} = \frac{\mu^2 c_1}{\rho a^3}\left(c_1 = -\frac{\partial p}{\partial x}\right) \text{ and } \frac{\partial T}{\partial X} = \frac{(T_1 - T_0)}{a}c_2$$

$$\text{where } c_2 = \frac{\partial \theta}{\partial x}\mathrm{s} = \mathrm{S/a} \tag{8}$$

where a is some standard length and T_0, T_1 are prescribed temperatures at the bottom and top.

Momentum equation:

$$\frac{d^2u}{dy^2} - \alpha^2 u = -c_1 \tag{9}$$

Energy equation:

$$\frac{d^2\theta}{dy^2} = P_r c_2 u - E\left(\frac{du}{dy}\right)^2 \tag{10}$$

Together with the boundary conditions for velocity:

$$\mathrm{u}(0) = s\frac{du}{dy}\,|_{y=0} \text{ and } \frac{du}{dy}\,|_{y=h} = 0 \tag{11}$$

and for temperature:

$$\theta\,(0) = 0 \text{ and } \theta\,(h) = 1 \tag{12}$$

3. Solution of the Problem

The transformed basic (9) and (10) together with the boundary conditions (11) and (12) are solved by applying Galerkin method of the solution first for the velocity profile and temperature, applying which the other fields are calculated and are given below as:

The velocity distribution:

$$u\,(y) = Bc_1\left(s + y - \frac{y^2}{2h}\right) \tag{13}$$

where $B = \dfrac{\left(s + \frac{h}{3}\right)}{\left(\frac{s}{h} + s^2\alpha^2 + \frac{2}{3}sh\alpha^2 + \frac{2}{5}\alpha^2h^2 + \frac{1}{3}\right)}$

Mean velocity:

$$\bar{u} = Bc_1\left(s + \frac{h}{3}\right) \tag{13a}$$

The temperature distribution:

$$\theta\,(y) = \frac{\Pr c_1c_2B}{24}\left[y^2\left(12s + 4y - \frac{y^2}{h}\right) - yh\,(12s + 3)\right] - \frac{EB^2c_1^2}{12}\left[y^2\left(6 + \frac{y^2}{h^2} - \frac{4y}{h}\right) - 3yh\right] + \frac{y}{h} \tag{14}$$

Mean temperature:

$$\begin{aligned}\bar{\theta} &= \frac{1}{h}\int_0^h \theta\,(y)\,dy \\ &= \frac{\Pr c_1c_2Bh^2}{24}\left[-2s - \frac{27h}{10}\right] - \frac{EB^2c_1^2h^2}{12}\left[\frac{6}{5}\right] + \frac{h}{2}\end{aligned} \tag{15}$$

Heat transfer rate (Nusselt Number):
On the free surface:

$$\left.\frac{d\theta}{dy}\right|_{y=h} = \frac{\Pr c_1c_2B}{24}\left(12sh + 5h^2\right) - \frac{EB^2c_1^2h}{12} + \frac{1}{h} \tag{16}$$

On the bottom:

$$\left.\frac{d\theta}{dy}\right|_{y=0} = \frac{\Pr c_1c_2B}{24}\left(-12sh - 3h^2\right) + \frac{EB^2c_1^2h}{4} + \frac{1}{h} \tag{17}$$

Special Case: Fluid flow in a medium with small porosity i.e., flow for large values of α or small values of the porosity coefficient k$*$.

In the case when porosity parameter α is very large after a series of calculations we come across the values of different profiles as:

Velocity Profile:

$$u(y) = \frac{c_1\left(s+\frac{h}{3}\right)\left(s+y-\frac{y^2}{2h}\right)}{\alpha^2\left[\left(s+\frac{h}{3}\right)^2+\frac{h^2}{45}\right]} \tag{18}$$

Mean velocity:

$$\bar{u} = \frac{c_1\left(s+\frac{h}{3}\right)^2}{\alpha^2\left[\left(s+\frac{h}{3}\right)^2+\frac{h^2}{45}\right]} \tag{19}$$

Temperature:

$$\theta = \frac{\Pr c_1c_2}{24}\frac{\left(s+\frac{h}{3}\right)}{\alpha^2\left(\left(s+\frac{h}{3}\right)^2+\frac{h^2}{45}\right)}\left[y^2\left(12s+4y-\frac{y^2}{h}\right)-yh\left(12s+3\right)\right]$$
$$-\frac{Ec_1^2}{12}\left[\frac{\left(s+\frac{h}{3}\right)}{\alpha^2\left(\left(s+\frac{h}{3}\right)^2+\frac{h^2}{45}\right)}\right]^2\left[y^2\left(6+\frac{y^2}{h^2}-\frac{4y}{h}\right)-3yh\right]+\frac{y}{h} \tag{20}$$

Mean temperature:

$$\bar{\theta} = \frac{\Pr c_1c_2h^2\left(s+\frac{h}{3}\right)}{24\alpha^2\left(\left(s+\frac{h}{3}\right)^2+\frac{h^2}{45}\right)}\left[-2s-\frac{27h}{10}\right]-\frac{Ec_1^2h^2}{12}\left[\frac{\left(s+\frac{h}{3}\right)}{\alpha^2\left(\left(s+\frac{h}{3}\right)^2+\frac{h^2}{45}\right)}\right]^2\left[\frac{6}{5}\right]+\frac{h}{2} \tag{21}$$

Heat transfer rate (Nusselt Number):
On the free surface:

$$\left.\frac{d\theta}{dy}\right|_{y=0} = \frac{\Pr c_1c_2}{24}\frac{\left(s+\frac{h}{3}\right)}{\alpha^2\left(\left(s+\frac{h}{3}\right)^2+\frac{h^2}{45}\right)}\left(-12s-3h^2\right)+\frac{Ec_1^2h}{4}\left[\frac{\left(s+\frac{h}{3}\right)}{\alpha^2\left(\left(s+\frac{h}{3}\right)^2+\frac{h^2}{45}\right)}\right]^2+\frac{1}{h} \tag{22}$$

On the bottom:

$$\left.\frac{d\theta}{dy}\right|_{y=h} = \frac{\Pr c_1c_2}{24}\frac{\left(s+\frac{h}{3}\right)}{\alpha^2\left(\left(s+\frac{h}{3}\right)^2+\frac{h^2}{45}\right)}\left(12sh+5h^2\right)-\frac{Ec_1^2h}{12}\left[\frac{\left(s+\frac{h}{3}\right)}{\alpha^2\left(\left(s+\frac{h}{3}\right)^2+\frac{h^2}{45}\right)}\right]^2+\frac{1}{h} \tag{23}$$

4. Results and Discussions

It is evident from Fig. 2 that the velocity of the flow in the channel uniformly increases with the increasing smaller values of the slip parameter s. It is noticed that the velocity profiles are more steep for large values of α that is the velocity of the fluid decreases with the increase in the value of α (Fig. 8).

It is evident from the Fig. 3 that for the increasing values of the pressure gradient c_1 the mean velocity increases and appears to be decreasing with the increase in the values of α. Fig. 9 (for the case of large α), illustrates that the mean velocity is high for the larger values of the slip parameter s.

It is evident from Fig. 4 for a fixed slip the temperature increases with the increasing values of α where as in the case of large α the temperature slightly decreases with the increasing values of the Prantdl number Pr (Fig. 10).

From Fig. 5 it is clear that for increasing slips the mean temperature increases and in the case of large $\alpha$the mean temperature decreases with the increasing values of the Prandtl number Pr (Fig. 11).

At the free surface the heat transfer rate increases with the increasing values of the slip parameter s (Fig. 6) where as in the case of large α for the increasing values of the porosity parameter the heat transfer rate on the bottom increases (Fig. 12).

The heat transfer rate at the bottom plate decreases with the increasing values of the slip s (Fig. 7) and in the case of large α heat transfer rate decreases with the increasing values of the porosity (Fig. 13).

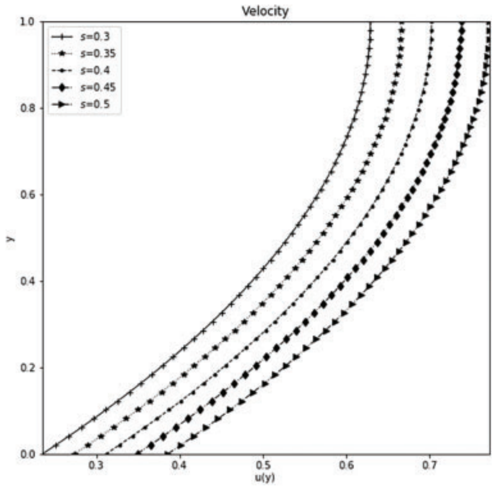

Fig. 2. Velocity profile for $c_1 = 1$, $h = 1$, and $\alpha = 0.5$.

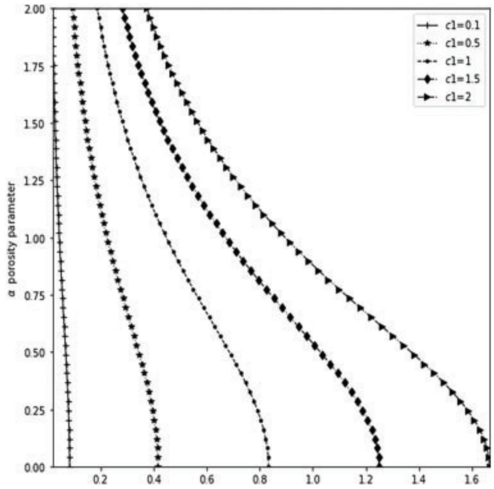

Fig. 3. Mean velocity for $h = 1$ and $s = 0.5$.

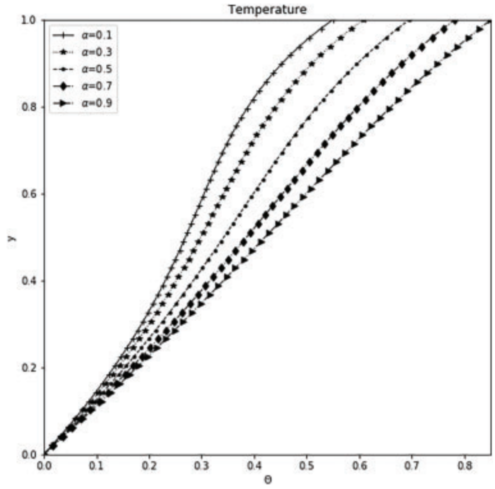

Fig. 4. Temperature distribution for $Pr = 0.1$, $c_1 = 1$, $E = 1$, $h = 1$, and $s = 0.5$.

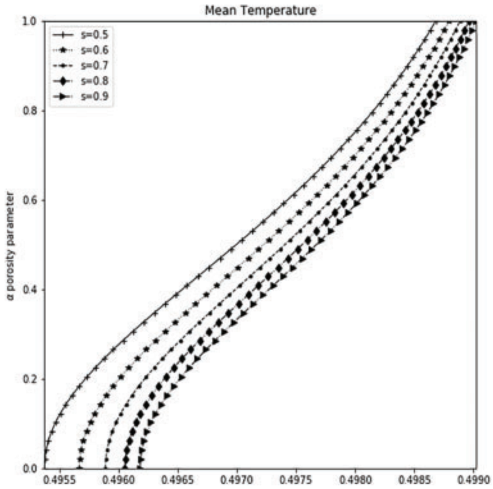

Fig. 5. Mean temperature distribution for $Pr = 0.1$, $c_1 = 1$, $c_2 = 0.5$, $E = 1$, $h = 1$.

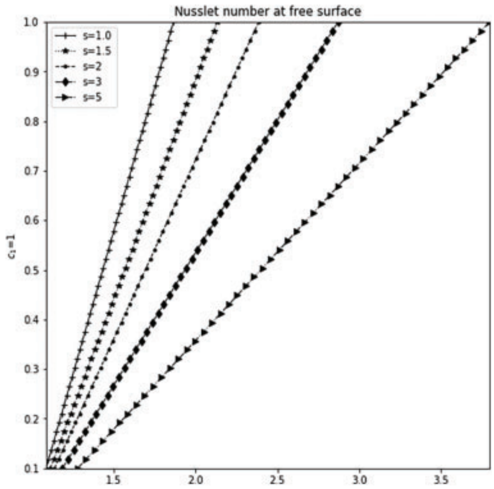

Fig. 6. Nusslet number for $Pr = 0.1$, $c_2 = 1$, $E = 1$, $h = 1$.

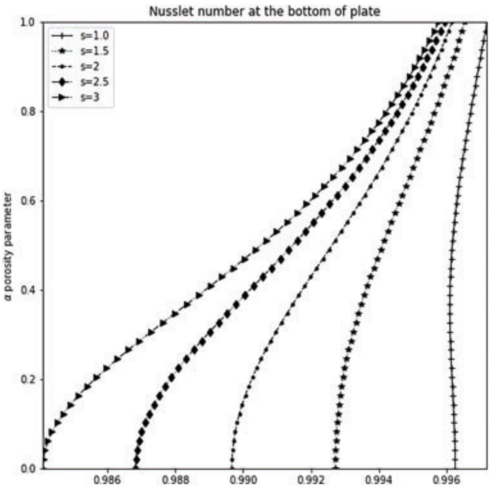

Fig. 7. Nusslet number for $Pr = 0.1$, $c_2 = 1$, $c_1 = 1$, $E = 1$, $h = 1$.

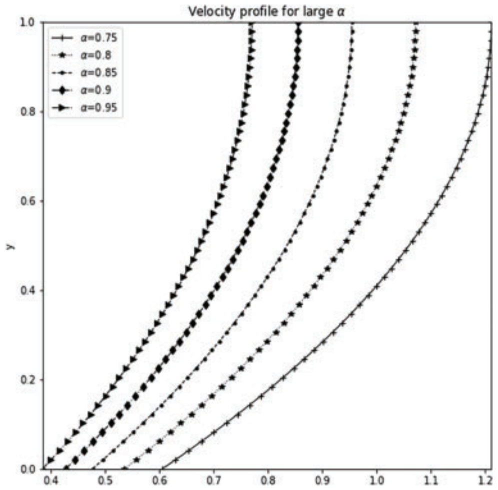

Fig. 8. Velocity profile for α when slip s = 0.5 and h = 1.

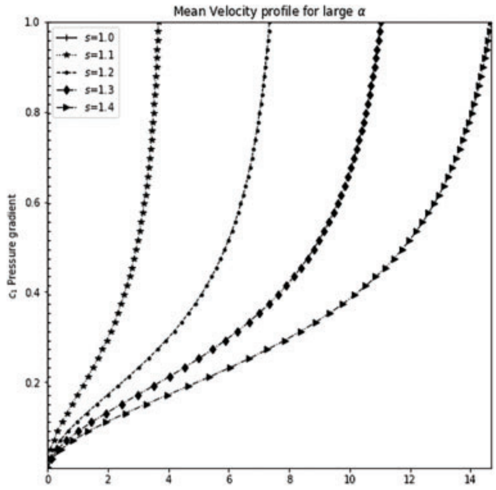

Fig. 9. Mean velocity profile for large α when h = 1.

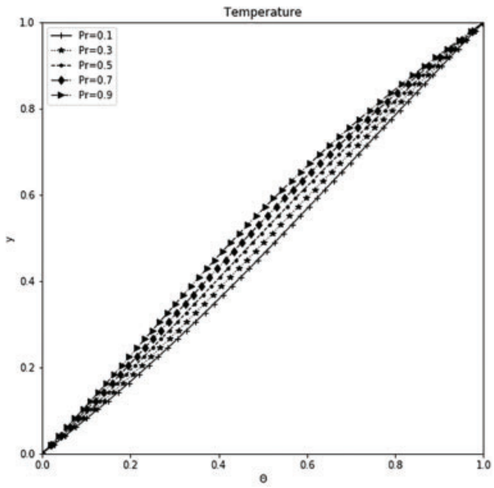

Fig. 10. Temperature distribution for large α $c_1 = 1$, $c_2 = 1$, E = 1, h = 1, and s = 0.5.

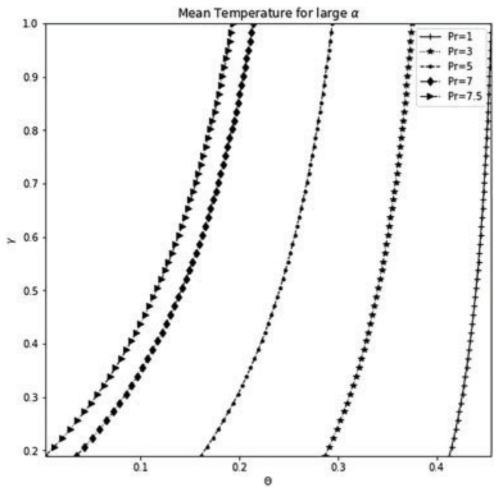

Fig. 11. Mean temperature distribution for large α $c_1 = 1$, $c_2 = 1$, E = 1, h = 1, and s = 0.5.

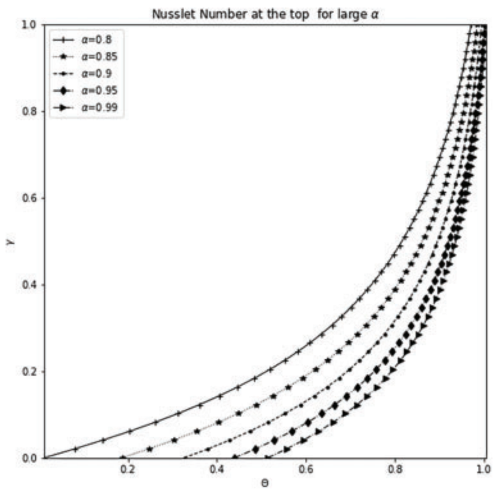

Fig. 12. Nusslet number at top of the large α $c_1 = 1$, $c_2 = 1$, E = 1, h = 1.

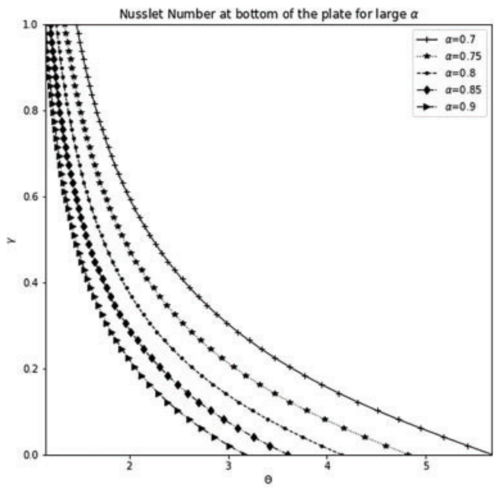

Fig. 13. Nusslet number at bottom of the large α $c_1 = 0.1$, $c_2 = 1$, E = 1, h = 1.

Conflict of Interest

Authors declare that they do not have any conflict of interest.

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