A New Formulation of a Set of Even Numbers

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***Abstract* — We present a new definition of an Even number as an integer, *E*, of the form $E=(n_1+n_2)+(n_2-n_1)^n$, $\forall n_1,n_2,n\in N$. We have shown the new set representation of even numbers holds for all natural numbers. This new representation opens up new doors to the solution of the Strong Goldbach Conjecture. The proofs obtained here will have impressive application in partitioning a given even number into all pairs of odd numbers.**

***Keywords* — Even numbers, Goldbach Conjecture, Natural numbers, Odd numbers, Prime numbers.**

I. Introduction

Even numbers are important in many areas of mathematics and computer science and are often used in algorithms and data structures [1], they play an important role in many mathematical concepts, such as algebra, number theory, and geometry. They are also commonly used in everyday life, such as when dividing objects into equal parts [2]. The set of even numbers is denoted by the symbol "E", and is a subset of the set of integers. Note that the set of even numbers includes both positive and negative integers, as well as zero. They have many interesting properties, such as the fact that any even number can be expressed as the sum of two prime numbers (known as the Strong Goldbach's conjecture) [3]. An easy way to identify even numbers is to check if the last digit belongs to the set [0, 2, 4, 6, 8] [4]. For example, 132, 4620,164 and 8888 are all even numbers because their last digit belongs to this set. In general, any even number can be represented as $2n$, where n is an integer.

II. Basic Mathematical Concepts

We introduce important known mathematical concepts that cover the basic aspects of numbers. We will use this understanding, to discuss and prove the new representation of even numbers. Properties of even numbers can be helpful in solving mathematical problems and understanding patterns in numbers. Some interesting properties that even numbers possess include:

Statement (1): Any even number added to another even number will always give an even number [5].

Statement (2): For all odds, when adding an odd and an odd number the sum will always be even [6].

The following *Theorem A* proves *statement (2):* It will help us understand the proof that will be made for the new set representation of even numbers when the natural numbers are odd.

A. Theorem

The sum of two odd numbers is an even number.

Proof (CASE 1)

A number is odd if its rightmost digit belongs to the set [1, 3, 5, 7, 9] while a number is even if its rightmost digit belongs to the set [0, 2, 4, 6, 8]. To find the rightmost digit of the sum of two numbers, you only have to add the rightmost digits of the two numbers and take the rightmost digit of the sum. For example, consider the numbers 1345 and 629. The rightmost digits are 5 and 9. Adding these gives us 14, whose rightmost digit is 4. So, we expect the rightmost digit of 1345 + 629 to be 4. And it is 1345 + 629 = 1974. This tells us that in order to verify that the sum of any two odd numbers is an even number, we just have to check whether the sum of any two odd digits has an even digit on the right. We have gone from talking about all the odd numbers (infinity of them) to talking about just five digits. We just checked this criterion for 5 and 9. We have to go through every case so that we are sure it always works:

1+1=2   3+1=4   …   9+1=10

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1+3=4	3+3=6	…	9+3=12
1+5=6	3+5=8	…	9+5=14
1+7=8	3+7=10	…	9+7=16
1+9=10	3+9=12	…	9+9=18

In every single one of these cases, the rightmost digit is even [7].

Proof (CASE 2)

A number is odd if it can be written as 2x + 1, where x is some integer (The word integer means whole number, either positive, negative, or zero). A number is even if it can be written as 2x, where x is some integer. To start, pick any two odd numbers. We can write them as 2n + 1 and 2m + 1. The sum of these two odd numbers is (2n + 1) + (2m + 1). This can be simplified to 2n + 2m + 2 and further simplified to 2(n + m + 1). The number 2(n + m + 1) is even because n + m + 1 is an integer. Therefore, the sum of the two odd numbers is even [7].

Statement (3): when adding an even number and an odd number the sum will always be odd [6]. We introduce *Theorem B* to prove this statement.

B. Theorem

The sum of any even number and any odd number is odd.

Proof

Let the even number be 2n and the odd number be 2m + 1. Adding gives 2n + (2m + 1) = 2n + 2m + 1 = 2 (n + m) +1. Thus, the expression can be written as one more than a multiple of 2, so it is odd [9].

Statement (4): Every integer is either even or odd (i.e., there are no other possibilities).

C. Relationship Between Even and Prime Numbers

There is a unique relationship between even and prime numbers: the only even prime number is 2. All other even numbers are divisible by 2 and therefore not prime. This relationship arises because a prime number is defined as a positive integer greater than 1 that has no positive integer divisors other than 1 and itself. However, any even number greater than 2 is divisible by 2 and therefore has at least two positive integer divisors (1 and 2). This means that no even number greater than 2 can be prime.

III. Formulation of a Set of Even Numbers

Euclid defined an even number as "a number which is divisible into two equal parts''. He also provided a method for generating even numbers using the formula $2n$, where n is any integer, and the representation of even numbers became more standardized. We explore this definition of even numbers ($2n$) and partition it into two even partitions say $2n = 2n_1 + 2n_2$. Further, we partition each of the even partitions into two such that $2n_1 = k_1 + k_2$ and $2n_2 = k_2 - k_1, \forall k_1, k_2 \in N$, where N is the set of all natural numbers. We, therefore, wish to define a new representation of even numbers E as, $E = (k_1 + k_2) + (k_2 - k_1)^n \; \forall k_1, k_2, n \in N$.

We introduce this new representation of even numbers and show that the new formulation will always give an even number. We prove this new representation, $E = (k_1 + k_2) + (k_2 - k_1)^n \; \forall k_1, k_2, n \in N$ is even in three cases: (1) when $k_1, k_2 \in 2n$, (2) when $k_1, k_2 \in 2n+1$, and (3) when $k_1, k_2 \in p$, where n is any natural number and P the set of all prime numbers. We introduce expression (1) which will be the general representation of the new formulation of sets of even numbers.

A. The New Representation of Even Numbers

Let n_1 and $n_2 \in N$, then $(n_1 + n_2) + (n_2 - n_1)^n$ is even for all $n_2 > n_1$ and $(n \geq 1) \in N$ (1)
where N is the set of all natural numbers Remark (for Expression (1)).

Since n_1 and $n_2 \in N$ then n_1 and $n_2 \in N$ can either be odd or even. Expression (1) will first be proved for even values of n_1 and $n_2 \in N$ and finite values of $n = 1{,}2$ and then generalized for all even values of n_1 and n_2.

CASE 1

This section will introduce *Corollary C and D* and provide proof for the new formulation of even numbers for finite values of $n = 1$ and 2 and generalize the proof for all values of n.

B. Corollary

Let n_1 and $n_2 \in 2k$, $\forall$ $k \in N$ and $n = 1$, then $(n_1 + n_2) + (n_2 - n_1)^1$ is even. (2)

Proof

Since n_1 and n_2 are even, then from the definition of even numbers they can be written as $n_1 = 2k_1, \forall k_1 \in N$ and $n_2 = 2k_2, \forall k_2 \in N$. Substituting $n_1 = 2k_1$ and vin expression (2) yields $(2k_1 + 2k_2) + (2k_2 - 2k_1)^1$. Since $n_2 > n_1$ then it implies that $k_2 > k_1 \Rightarrow \quad k_2 - k_1 > 0$.

$\Rightarrow 2(k_1 + k_2) + 2(k_2 - k_1)^1$ and from the definition of even numbers, both $2(k_1 + k_2)$ and $2(k_2 - k_1)$ are even hence $2(k_1 + k_2) + 2(k_2 - k_1)^1$ is even.

C. Corollary

Let n_1 and $n_2 \in 2k$, $\forall$ $k \in N$ and $n = 2$, then $(n_1 + n_2) + (n_2 - n_1)^2$ is even. (3)

Proof

Substituting $n_1 = 2k_1, \forall k_1 \in N$ and $n_2 = 2k_2, \forall k_2 \in N$ in expression (3) yields $(2k_1 + 2k_2) + (2k_2 - 2k_1)^2$ $\Rightarrow (2k_1 + 2k_2) + (2k_2 - 2k_1)^2 \Rightarrow 2(k_1 + k_2) + (4k_2^2 - 8k_2k_1 + 4k_1^2) \Rightarrow 2(k_1 + k_2) + 4(k_2^2 - 2k_2k_1 + k_1^2)$

$\Rightarrow 2(k_1 + k_2) + 4(k_2 - k_1)^2 \Rightarrow 2(k_1 + k_2) + 2^2(k_2 - k_1)^2$

$\Rightarrow 2(k_1 + k_2)$ is even and $2^2(k_2 - k_1)^2 = 2(2(k_2 - k_1)^2)$, for $k_2 - k_1 > 0 \Rightarrow (k_2 - k_1)^2 \in N$ and $2(k_2 - k_1)^2$ is even from the definition of even numbers. This further implies that $2(2(k_2 - k_1)^2)$ is also even and finally, this shows that $(2k_1 + 2k_2) + (2k_2 - 2k_1)^2$ is even.

D. Theorem

let n_1 and $n_2 \in 2k$,then $(n_1 + n_2) + (n_2 - n_1)^n$ is even for all $n_2 > n_1$ and $(n \geq 1) \in N$ (4)

Proof

From expressions (2) and (3) it is clear that $(n_1 + n_2)$ in expression (4) is even , and we are left to prove that $(n_2 - n_1)^n$ is also even.

$\Rightarrow (2k_2 - 2k_1)^n \Rightarrow 2^n(k_2 - k_1)^n = 2(2^{n-1}(k_2 - k_1)^n)$, where $(k_2 - k_1)^n \in N$

$\Rightarrow (2^{n-1}(k_2 - k_1)^n) \in N \Rightarrow 2(2^{n-1}(k_2 - k_1)^n)$ is even from the definition of even numbers. This further implies that $(2k_1 + 2k_2) + (2k_2 - 2k_1)^n$ is even.

CASE 2

This section will introduce *Corollary F and G* and provide proofs for the new representation of even numbers for odd values of $n_1 and\ n_2$ using values of $n = 1$ and 2, and then generalize the proof for all values of n.

E. Corollary

Let n_1 and $n_2 \in 2k + 1$ $\forall$ $k \in N$ and $n = 1$, then $(n_1 + n_2) + (n_2 - n_1)^1$ is even. (5)

Proof

Since n_1 and n_2 are both odd, from the definition of odd numbers, they can be written as $n_1 = 2k_1 + 1, \forall k_1 \in N$ and $n_2 = 2k_2 + 1, \forall k_2 \in N$. Substituting $n_1 = 2k_1 + 1$ and $n_2 = 2k_2 + 1$ in expression (5) yields $(2k_1 + 1 + 2k_2 + 1) + (2k_2 + 1 - 2k_1 - 1)^1$. Since $n_2 > n_1$ then it implies that $k_2 > k_1 \Rightarrow \quad k_2 - k_1 > 0$.

$\Rightarrow 2(k_1 + k_2 + 1) + 2(k_2 - k_1)^1$ and from the definition of even numbers, both $2(k_1 + k_2 + 1)$ for $(k_1 + k_2 + 1) \in N$ and $2(k_2 - k_1)$ are even, hence $2(k_1 + k_2 + 1) + 2(k_2 - k_1)^1$ is even.

F. Corollary

Let n_1 and $n_2 \in 2k + 1$ for $\forall$ $k \in N$ and $n = 2$, then $(n_1 + n_2) + (n_2 - n_1)^2$ is even (6)

Proof

Substituting $n_1 = 2k_1 + 1, \forall k_1 \in N$ and $n_2 = 2k_2 + 1, \forall k_2 \in N$ in expression (6) yields $(2k_1 + 2k_2 + 2) + (2k_2 - 2k_1)^2$

$\Rightarrow 2(k_1 + k_2 + 1) + 2^2(k_2 - k_1)^2 \Rightarrow 2(k_1 + k_2 + 1) + 4(k_2^2 - 2k_2k_1 + k_1^2) \Rightarrow$

$2(k_1 + k_2 + 1) + 2(2(k_2^2 - 2k_2k_1 + k_1^2)) \Rightarrow$ From the definition of even numbers, we have that $2(k_1 + k_2 + 1)$ and $2(2(k_2^2 - 2k_2k_1 + k_1^2))$ are both even proving expression (6).

G. Theorem

Let n_1 and $n_2 \in 2k+1$, then $(n_1+n_2)+(n_2-n_1)^n$ is even for all $n_2 > n_1$ and $(n \geq 1) \in N$ (7)

Proof

From expressions (5) and (6) it is clear that (n_1+n_2) in equation (7) is even. we are therefore left to prove that $(n_2-n_1)^n$ is also even.

$(2k_2-2k_1)^n \Rightarrow 2^n(k_2-k_1)^n = 2(2^{n-1}(k_2-k_1)^n)$, where $(k_2-k_1)^n \in N$

$\Rightarrow (2^{n-1}(k_2-k_1)^n) \in N \Rightarrow 2[(2^{n-1}(k_2-k_1)^n)]$ is even. This further implies that $(2k_2-2k_1)^n$ is even.

CASE 3

This section will give a brief overview of Prime numbers and introduce corollary *A.*

IV. Brief Overview of Prime Numbers

Every prime number is an odd number, except for 2, which is the only even prime number. However, not every odd number is a prime number. For example, 9 is an odd number, but it is not a prime number because it can be divided by 3.

The study of prime numbers and related problems such as the Goldbach Conjecture is an active area of research in mathematics and has important applications in Cryptography, Coding Theory, and Computer Science. Prime numbers are important in mathematics and computer science because they are used in cryptography, which is the study of codes and ciphers [8]. Odd numbers, on the other hand, are more commonly used in basic arithmetic and algebra.

A. Corollary

Prime numbers greater than 2 are subsets of odd numbers and therefore the results obtained in theorem *H* covers both composite odd numbers and primes. The theorem therefore proves expression (8).

Let p_1 and $p_2 \in P$,then $(p_1+p_2)+(p_2-p_1)^n$ is even $\forall n \in N$ and $p_2 > p_1$ (8)

This expression will have remarkable application in generating all pairs of odd numbers associated with a given even number of form in expression (8). These results will be used to investigate the possibility of obtaining at least one Goldbach partition from these pairs of odd numbers.

V. Conclusion

The new formulation of sets of even numbers has been proved in three cases: Case 1 asserts that the new representation of even numbers holds for all even natural numbers. For Case 2, this formulation of even numbers will always be even for all odd natural numbers. Since prime numbers greater than 2 are subsets of odd numbers, Case 3 gives an expression of the formulation of even numbers for all prime natural numbers. This new formulation of the form in expression (8) can be used to investigate the possibility of partitioning a given even number into all pairs of odd numbers. We expect the results obtained here to have an impact to finding the solution to the Strong Goldbach Conjecture.

VI. Conflict of Interest

Authors declare that they do not have any conflict of interest.

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