Detection and Rarefaction of the Twin Primes Numbers

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***Abstract* — In this manuscript are considered 3 types of numbers:**
a) integral numbers like for example (x)=10^10
b) prime numbers whose properties is to be only divisible by themselves
c) twin numbers
The number of twin primes contained under the number (x) is here derived by:
1) a mathematical function proposed by Gauss (1792-1796) based on a converging logarithmic sum,
2) Euclid's theorems on prime numbers.

***Keywords* —Prime Numbers; Gauss; Euclid; Theorem of arithmetic.**

I. Introduction

The Gauss observed that each time the number of primes increased by 10 in the first column, will be doubled in the second, and he recognized that this behavior was typical of the logarithms. Gauss exposed his observation on a table where he noticed that more the number (x) was big, more the primes associated with it were rarefied.

Gauss was also aware of the existence of the fundamental theorem of arithmetic with which Euclid demonstrated that there are infinite prime numbers.

Table I: Gauss Observation

(x)	twin pair	number of primes	
10^3	35	168	= 1.521739
10^4	205	1229	= 1.433356
10^5	1224	9592	= 1.330993
10^6	8169	78498	= 1.335676
10^7	58980	664579	= 1.335658
10^8	440312	5761455	= 1.323156
10^9	3424506	50847534	= 1.320881
10^10	27412679	455052511	= 1.320581
10^11	224376048	4118054813	= 1.323204
10^12	1870585220	37607912018	= 1.320377
10^13	15834664872	346065536839	= 1.320341
10^14	135780321665	3204941750802	= 1.320328
10^15	1177209242304	29844570422669	= 1.320326

II. Discussion

Taking into account what has been described so far, to determine the number of twin primes below the number (x), I consider the problem as a calculation of probability and I use the expression proposed by Gauss based on the inverse of a sum of logarithm, that is:

1) $1/\log 1+1/\log 2+1/\log 3+1/\log 4+ \quad =\sum(1/\log)$

It is important to notice that this expression proposed by Gauss provides a very quick converging function and allows to detect the prime numbers and consequently the twin.

Taking a casual decade of numbers contained between

2) $10n-4 \leq 10n \leq 10n+5$

It is knows that the primes can only be numbers ending with 1, 3, 7, 9, (like for example 11, 13, 17, 19, 29, 31, 41, 43…).

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In each decade only 4 types of numbers have a probability 1/log x to be prime. Thus in each decade, if a number is randomly drawn, it has a probability to be prime

3) 4 /10^ (1/log x).

One still needs to evaluate the probability that two numbers selected among those ending by 7, 9, 1, 3 form a pair (7-9, 9-1, 1-3)

We have

4 /10 ^1/logx +3/9/1/logx = 4/10 +1/3) ^logx

To prove this, it is sufficient to consider Euclid's theorems who demonstrate that each prime is given

4) 6m-1 or 6m+1.

However the primes can only be numbers terminating by 1, 3, 7, 9, like for example 11, 13, 17, 19, 29, 31, 41, 43.

Therefore using Euclid's theorems I can only have:

5) 6m-1 or 6m+1.

for example, 17 = m 6*3-1 = 18-1 or 19 = 6*3+1 = 18+1.

III. Conclusion

The number of twin primes is derived by Gauss observations and the support of the theory of probability associated with the use of Euclid's theorems.

References

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