On Fibonacci Cordial Labeling of Some Snake Graphs

Jolina E. Sulayman and Ariel C. Pedrano

***Abstract* —Let an injective function f from vertex set V of a graph G to the set $F_0, F_1, F_2, \dots, F_n$, where F_j is the jth Fibonacci number $(j = 0, 1, \dots, n)$, is said to be Fibonacci cordial labeling if the induced function f^* from the edge set E of graph G to the set $\{0, 1\}$ defined by $f^*(uv) = (f(u) + f(v)) \pmod 2$ satisfies the condition $|e_f(0) - e_f(1)| \leq 1$, where $e_f(0)$ is the number of edges with label 0 and $e_f(1)$ is the number of edges with label 1. A graph which admits Fibonacci cordial labeling is called Fibonacci cordial graph.**

***Keywords* —Graph labeling; Cordial labeling, Fibonacci Cordial Labeling.**

I. Introduction

All graphs in this paper are finite, simple and undirected. For various graph theoretic notations and terminology we follow Gross and Yellen [1]. A graph labeling is the assignment of labels, usually represented by an integer, to the vertices or edges or both of a graph. Labeling of graphs plays an important role in the field of graph theory because of its diversified and rigorous application such as design and analysis of communication networks, military surveillance, social sciences, optimization, Neutral Networks, Coding Theory, and Circuit Analysis and etc. In most applications, labels are positive or non-negative integers [2].

In this paper, [3] introduce Fibonacci cordial labeling. Assume G to be a simple connected graph with n vertices. An injective function f from vertex set V of a graph G to the set $F_0, F_1, F_2, \dots, F_n$, where F_j is the jth Fibonacci number $(j = 0, 1, \dots, n)$, is said to be Fibonacci cordial labeling if the induced function f^* from the edge set E of graph G to the set 0, 1 defined by $f^*(uv) = (f(u) + f(v))(mod\ 2)$ satisfies the condition $|e_f(0) - e_f(1)| \leq 1$, where $e_f(0)$ is the number of edges with label 0 and $e_f(1)$ is the number of edges with label 1. A graph which admits Fibonacci cordial labeling is called Fibonacci cordial graph. In this paper we discuss the Fibonacci cordial labeling.

II. Basic Concepts

Definition 2.1. An alternate triangular snake graph $A(T_n)$ obtained from a path $v_1, v_2, v_3, \dots, v_n$ by joining v_i and v_{i+1} (alternately) to new vertex u_i. That is every alternate edge of a path is replaced by C_3.

Definition 2.2. A quadrilateral Snake Graph Q_n is obtained from a path $v_1, v_2, \dots, v_n$ by joining v_i and v_{i+1} to two new vertices u_i and w_i for $1 \leq i \leq n-1$ respectively and then joining u_i and w_i.

Definition 2.3. A double alternate quadrilateral snake graph $DA(QS_n)$ consists of two alternate quadrilateral snakes that have a common path. That is, a double alternate quadrilateral snake is obtained from a path $v_1, v_2, v_3, \dots, v_n$ by joining v_i and v_{i+1} (alternately) to two new vertices u_i, x_i and w_i, y_i respectively and then joining u_i, x_i and w_i, y_i.

Definition 2.4. The cycle quadrilateral snake graph CQ_n with $q = 4n$ edges is a graph obtained from the cycle C_n by identifying each edge of C_n with an edge of C_4.

III. Results and Discussions

Theorem 3.1. [4] Let f_i be the ith term of the Fibonacci sequence. Then, for each $n \in \mathbb{N}$, f_{3n} is even, and all of the other terms in the Fibonacci sequence are odd.

Theorem 3.2. The Alternate Triangular Snake Graph $A(T_n)$ admits fibonacci cordial labeling for all $n \geq 2$.

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J. E. Sulayman, University of Southeastern Philippines, Philippines.
(corresponding e-mail: sulaymanjolina.98@gmail.com)
A. C. Pedrano, University of Southeastern Philippines, Philippines.
(e-mail: ariel.pedrano@usep.edu.ph)

Proof. Let $A(T_n)$ be an alternate triangular snake graph obtained from a path $v_1, v_2, v_3, \ldots, v_n$ by joining v_i and v_{i+1} (alternately) to new vertex u_i where $1 \leq i \leq \frac{n}{2}$ if n is even (Figure 1) and $1 \leq i \leq \frac{n-1}{2}$ if n is odd (Fig 2).

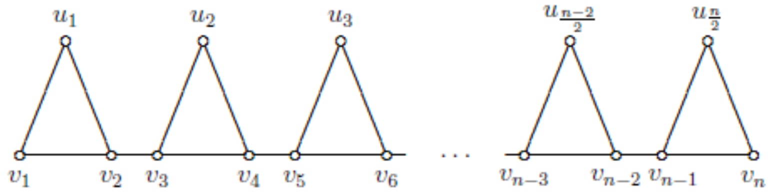

Fig. 1. An alternate triangular snake graph $A(T_n)$.

To prove the theorem, we will consider the following cases:
Case 1: n is even, $n \geq 2$.
The order and size of the alternate triangular snake graph is $A(T_n)$
$|V(A(T_n))| = \frac{3n}{2}$ and $|E(A(T_n))| = 2n - 1$,
respectively.
Define a function $f: V(A(T_n)) \to \left\{F_0, F_1, F_2, \ldots, F_{\frac{3n}{2}}\right\}$ by:

$$f(u_i) = F_{3(i-1)}, \ 1 \leq i \leq \frac{n}{2}$$
$$f(v_i) = \begin{cases} F_{\frac{3i-1}{2}}, & i = 1, 3, 5, \ldots, n-1 \\ F_{\frac{3i-2}{2}}, & i = 2,4,6, \ldots, n \end{cases}$$

By using Theorem 3.1, the edges of $A(T_n)$with labels zero and one are the following:
For $1 \leq i \leq \frac{n}{2}$, we have

$$f^*(v_{2i-1}u_i) = 1$$
$$f^*(v_{2i}u_i) = 1$$

For $1 \leq i \leq n-1$, we have

$$f^*(v_i v_{i+1}) = 0$$

In view of the above labeling, we have,

$$e_f(0) = n - 1 \text{ and } e_f(1) = \frac{n}{2} + \frac{n}{2} = n$$

Hence, $|e_f(0) - e_f(1)| = |(n-1) - n| = |-1| = 1 \leq 1$. Thus, the alternate triangular snake graph $A(T_n)$ is a fibonacci cordial graph if n is even, $n \geq 2$.
Case 2: n is odd, $n \geq 3$.

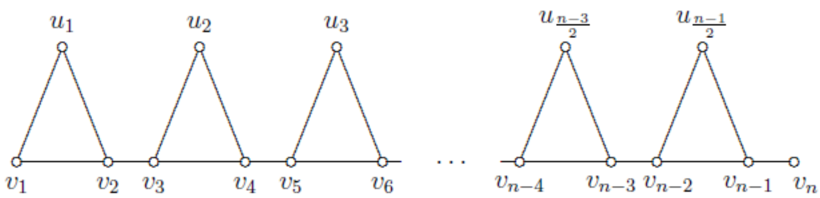

Fig. 2. An alternate triangular snake graph $A(T_n)$.

The order and size of the alternate triangular snake graph $A(T_n)$ is

$$|V(A(T_n))| = \frac{3n-1}{2} \text{ and } |E(A(T_n))| = 2n - 2,$$

respectively.
Define a function $f: V(A(T_n)) \to \left\{F_0, F_1, F_2, \ldots, F_{\frac{3n-1}{2}}\right\}$ by:

$$f(u_i) = F_{3(i-1),} 1 \leq i \leq \frac{n}{2}$$
$$f(v_i) = \begin{cases} F_{\frac{3i-1}{2}}, & i = 1, 3, 5, \dots, n \\ F_{\frac{3i-2}{2}}, & i = 2,4,6, \dots, n-1 \end{cases}$$

By using Theorem 3.1, the edges of $A(T_n)$with labels zero and one are the following:
For $1 \leq i \leq \frac{n-1}{2}$, we have

$$f^*(v_{2i-1}u_i) = 1$$
$$f^*(v_{2i}u_i) = 1$$

For $1 \leq i \leq n-1$, we have

$$f^*(v_i v_{i+1}) = 0$$

In view of the above labeling, we have,

$$e_f(0) = n-1 \text{ and } e_f(1) = \frac{n-1}{2} + \frac{n-1}{2} = n-1$$

Hence, $|e_f(0) - e_f(1)| = |(n-1) - (n-1)| = 0 \leq 1$. Thus, the alternate triangular snake graph $A(T_n)$ is a fibonacci cordial graph if n is odd, $n \geq 3$.

Considering the cases above, we could say that, the alternate triangular snake graph $A(T_n)$ is a fibonacci cordial graph for all $n \geq 2$. ■

Theorem 3.3. The Quadrilateral Snake Graph (Q_n) admits fibonacci cordial labeling for all $n \geq 2$.

Proof. Let Q_n be a quadrilateral snake graph obtained from a path $v_1, v_2, v_3, \dots, v_n$ by joining v_i and v_{i+1} to new vertices u_i and w_i where $1 \leq i \leq n-1$ for all $n \geq 2$ (See Fig. 3).

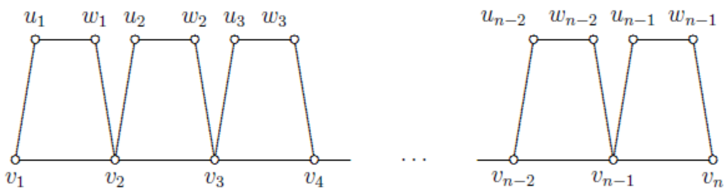

Fig. 3. A quadrilateral snake graph Q_n.

The order and size of the quadrilateral snake graph Q_n is

$$|V(Q_n)| = 3n - 2 \text{ and } |E(Q_n)| = 4n - 4,$$

respectively.

Define a function $f: V(Q_n) \to \{F_0, F_1, F_2, \dots, F_{3n-2}\}$ by:

$$f(v_i) = F_{3(i-1),} \quad 1 \leq i \leq n$$
$$f(u_i) = F_{3i-2,} \quad 1 \leq i \leq n-1$$
$$f(w_i) = F_{3i-1,} \quad 1 \leq i \leq n-1$$

By using Theorem 3.1, the edges of Q_n with labels zero and one are the following:
For $1 \leq i \leq n-1$, we have

$$f^*(v_i v_{i+1}) = 0$$
$$f^*(u_i w_i) = 0$$
$$f^*(v_i u_i) = 1$$
$$f^*(v_{i+1} w_i) = 1$$

In view of the above labeling, we have,

$$e_f(0) = n - 1 + n - 1 = 2n - 2$$

and

$$e_f(1) = n - 1 + n - 1 = 2n - 2$$

Hence, $|e_f(0) - e_f(1)| = |(2n - 2) - (2n - 2)| = 0 \leq 1$. Thus, the quadrilateral snake graph Q_n is a fibonacci cordial graph for all $n \geq 2$. ■

Theorem 3.4 The Double Alternate Quadrilateral Snake Graph $(DA(QS_n))$ admits fibonacci cordial labeling for all $n \geq 2$.

Proof. Let$(DA(QS_n))$ be a double alternate quadrilateral snake graph obtained from a path $v_1, v_2, v_3, \ldots, v_n$ by joining v_i and v_{i+1} (alternately) to two new vertices $u_i,\ x_i$ and $w_i,\ y_i$ respectively and then joining $u_i,\ x_i$ and $w_i,\ y_i$ where $1 \leq i \leq \frac{n}{2}$ if n is even (Figure 4) and $1 \leq i \leq \frac{n-1}{2}$ if n is odd (Fig. 5).

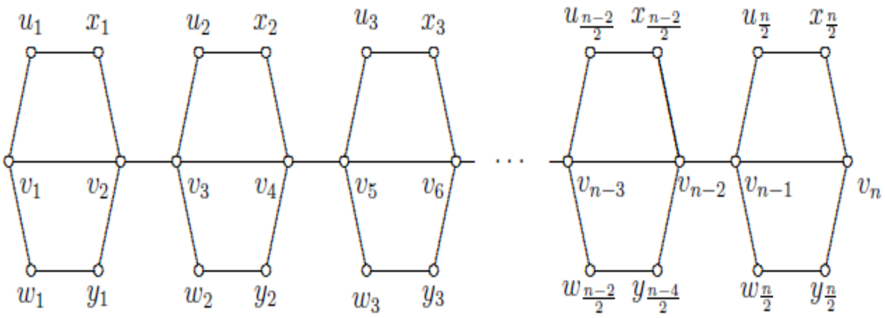

Fig. 4. Double Alternate Quadrilateral Snake Graph $DA(QS_n)$.

To prove the theorem, we will consider the following cases:

Case 1: n is even, $n \geq 2$.

The order and size of the double alternate quadrilateral snake graph $(DA(QS_n))$ is

$$|V(DA(QS_n))| = 3n \text{ and } |E(DA(QS_n))| = 4n - 1,$$

respectively.

Define a function $f: V(DA(QS_n)) \to \{F_0, F_1, F_2, \ldots, F_{3n}\}$ by:

$$f(v_i) = F_{3i-3}, \quad 1 \leq i \leq n$$
$$f(u_i) = F_{6i-5}, \quad 1 \leq i \leq \frac{n}{2}$$
$$f(x_i) = F_{6i-4}, \quad 1 \leq i \leq \frac{n}{2}$$
$$f(w_i) = F_{6i-2}, \quad 1 \leq i \leq \frac{n}{2}$$
$$f(y_i) = F_{6i-1}, \quad 1 \leq i \leq \frac{n}{2}$$

By using Theorem 3.1, the edges of $DA(QS_n)$ with labels zero and one are the following:

For $1 \leq i \leq n - 1$, we have

$$f^*(v_i v_{i+1}) = 0$$

For $1 \leq i \leq \frac{n}{2}$, we have

$$f^*(v_{2i-1}u_i) = 1; \qquad f^*(v_{2i}y_i) = 1;$$
$$f^*(v_{2i}x_i) = 1; \qquad f^*(u_i x_i) = 0;$$
$$f^*(v_{2i-1}w_i) = 1; \qquad f^*(w_i y_i) = 0;$$

In view of the above labeling, we have,

$e_f(0) = n - 1 + \frac{n}{2} + \frac{n}{2} = 2n - 1$

and

$$e_f(1) = \frac{n}{2} + \frac{n}{2} + \frac{n}{2} + \frac{n}{2} = 2n$$

Hence, $|e_f(0) - e_f(1)| = |(2n-1) - (2n)| = |-1| = 1 \leq 1$. Thus, the double alternate quadrilateral snake graph $DA(QS_n)$ is a fibonacci cordial graph if n is even, $n \geq 2$.

Case 2: n is odd, $n \geq 3$.

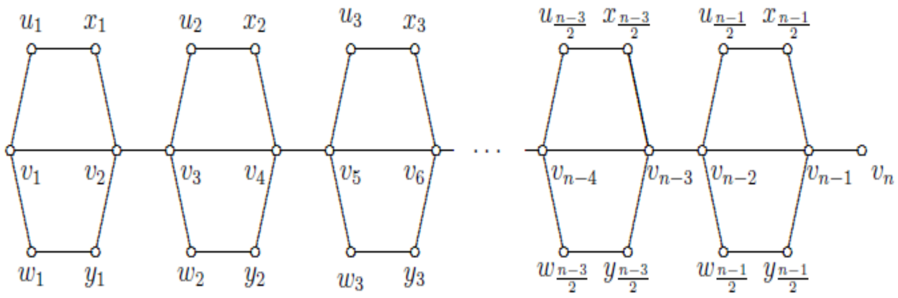

Fig. 5. Double Alternate Quadrilateral Snake Graph $DA(QS_n)$.

The order and size of the double alternate quadrilateral snake graph $(DA(QS_n))$ is

$$|V(DA(QS_n))| = 3n - 2 \text{ and } |E(DA(QS_n))| = 4n - 4,$$

respectively.

Define a function $f: V(DA(QS_n)) \to \{F_0, F_1, F_2, \ldots, F_{3n-2}\}$ by:

$$f(v_i) = F_{3i-3}, \quad 1 \leq i \leq n$$
$$f(u_i) = F_{6i-5}, \quad 1 \leq i \leq \frac{n-1}{2}$$
$$f(x_i) = F_{6i-4}, \quad 1 \leq i \leq \frac{n-1}{2}$$
$$f(w_i) = F_{6i-2}, \quad 1 \leq i \leq \frac{n-1}{2}$$
$$f(y_i) = F_{6i-1}, \quad 1 \leq i \leq \frac{n-1}{2}$$

By using Theorem 3.1, the edges of $DA(QS_n)$ with labels zero and one are the following:

For $1 \leq i \leq n-1$, we have

$$f^*(v_i v_{i+1}) = 0$$

For $1 \leq i \leq \frac{n-1}{2}$, we have

$$f^*(v_{2i-1}u_i) = 1; \qquad f^*(v_{2i}y_i) = 1;$$
$$f^*(v_{2i}x_i) = 1; \qquad f^*(u_i x_i) = 0;$$
$$f^*(v_{2i-1}w_i) = 1; \qquad f^*(w_i y_i) = 0;$$

In view of the above labeling, we have,

$$e_f(0) = n - 1 + \frac{n-1}{2} + \frac{n-1}{2} = 2n - 2$$

and

$$e_f(1) = \frac{n-1}{2} + \frac{n-1}{2} + \frac{n-1}{2} + \frac{n-1}{2} = 2n - 2$$

Hence, $|e_f(0) - e_f(1)| = |(2n-2) - (2n-2)| = 0 \leq 1$. Thus, the double alternate quadrilateral snake graph $DA(QS_n)$ is a fibonacci cordial graph if n is odd, $n \geq 3$.

Considering the cases above, we could say that, the double alternate quadrilateral snake graph $DA(QS_n)$ is a fibonacci cordial graph for all $n \geq 2$. ■

Theorem 3.5. The Cycle Quadrilateral Snake Graph CQ_n admits fibonacci cordial labeling for all $n \geq 3$.

Proof. Let CQ_n be a cycle quadrilateral snake graph obtained from the cycle C_n by identifying each edge of C_n with an edge of C_4 (See Fig. 6).

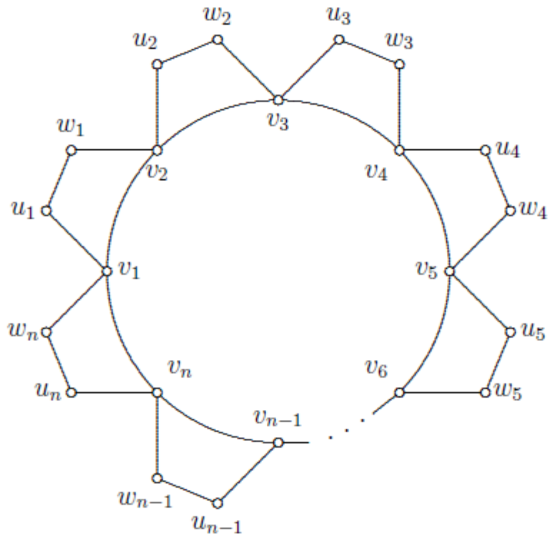

Fig. 6. A cycle quadrilateral snake graph CQ_n.

The order and size of the cycle quadrilateral snake graph CQ_n is

$$|V(CQ_n)| = 3n \text{ and } |E(CQ_n)| = 4n - 4,$$

respectively.
Define a function $f: V(CQ_n) \to \{F_0, F_1, F_2, \dots, F_{3n}\}$ by:

$$f(v_i) = F_{3(i-1)}, \quad 1 \le i \le n$$
$$f(u_i) = F_{3i-2}, \quad 1 \le i \le n$$
$$f(w_i) = F_{3i-1}, \quad 1 \le i \le n$$

By using Theorem 3.1, the edges of CQ_n with labels zero and one are the following:

$$f^*(v_1 v_n) = 0$$
$$f^*(v_1 w_n) = 1$$

For $1 \le i \le n - 1$, we have

$$f^*(v_i v_{i+1}) = 0$$
$$f^*(v_{i+1} w_i) = 1$$

For $1 \le i \le n$, we have

$$f^*(u_i w_i) = 0$$
$$f^*(v_i u_i) = 1$$

In view of the above labeling, we have,

$$e_f(0) = 1 + (n - 1) + n = 2n$$

and

$$e_f(1) = 1 + (n - 1) + n = 2n$$

Hence, $|e_f(0) - e_f(1)| = |2n - 2n| = 0 \le 1$. Thus, the cycle quadrilateral snake graph CQ_n is a fibonacci cordial graph for all $n \ge 3$. ■

Conflict of Interest

Authors declare that they do not have any conflict of interest.

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