Parametric Sensitivity Analysis of A Compressible Multiphase Flow Model in Porous Medium: Application to the Tsimiroro Madagascar Oil Reservoir

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***Abstract* — Modeling and numerical methods are two very important fields in physics and engineering sciences. In fluid mechanics, they allow us to study various complex problems and to make predictions of complex phenomena. However, in some cases like the field of petroleum engineering, many parameters like absolute permeability, relative permeability, porosity, capillary pressures, etc. are difficult to measure and / or estimate with certainty. The parametric sensitivity analysis of models provides an overview of the most influential parameters of a model and thus enables the model to be optimized. The study carried out in this work goes in this direction and has made it possible to identify the most influential parameters. The results obtained show that the most influential parameters of the model are the geometric characteristics of the reservoir, porosity and permeability, as well as the injection pressure in the wells.**

***Keywords* — Analysis of variance, multiphase flow, oil reservoir, porous media, sensitivity analysis.**

I. INTRODUCTION

In the fields of engineering sciences and physics, modeling is an essential step for understanding systems. The model complexity depends on the observed phenomena and the systems to be modeled, but in general physical models are complex. Uncertainties related to the input parameters of a model are among the causes that can make a model complex. Sensitivity analysis is proving to be a reliable tool for evaluating the relationship between inputs and outputs of a model or system. It is used in several areas of the engineering sciences for the optimization and evaluation of systems. In the literature there are many methods of sensitivity analysis, each of which has its advantages and disadvantages. Reference [1], present most of the global sensitivity analysis methods that are most used in the engineering sciences. Reference [2] use the FAST methods and the analysis of variance method to study the sensitivity of the input parameters of a model that simulates a natural convection decay heat removal system. The authors show that the use of variance analysis is more efficient if the distributions of the output parameters of the model are normal. The study by [3], combines the variance-based method and the RBD-FAST method to make an overall sensitivity analysis of a nonlinear mathematical model. The authors show that this combination is achievable only when a shared sampling strategy is used. To do this, the authors apply a radial design approach with Sobol's quasi-random sequences to generate a basic sample. The results of their study show that the method can be used for an overall sensitivity analysis of a nonlinear model without assuming any restriction on the model except that the model input parameters must be independent and uniformly distributed. Another interesting work carried out by [4], proposes a global sensitivity analysis tool called GOSAT for the analysis of complex models. The method implemented in this tool is a method derived from the FAST method, proposed by [5] and improved by [6].

Flow in porous media occupies a very important place in the field of energy, environment and process engineering. Studying physical problems related to these environments always requires very complex mathematical models with many uncertainties. Thus, the use of sensitivity analysis is necessary to study the influence of parameters on the output of such models. Reference [7] use a method based on singular value decomposition (SVD) to study the influence of the input parameters of a flow model in porous media in the case of nuclear waste disposal. The authors show that in comparison with statistical approaches, this method has a low computational cost and makes it possible to study the variability of the results of the

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sensitivity analysis due to variations in the input parameters of the model, and thus provide a criterion of quality for the validity of more comprehensive global probabilistic approaches. In their work, [8], carried out a sensitivity analysis study of a multiphase flow in porous geological media of hydrogen brine. Pore scale analysis was employed to predict the flow of hydrogen in storage formations but also to quantify the sensitivity of the micro-scale characteristics of the contact angle and porosity of the rock structure. Then, the sensitivity index was used for the individual impact of the parameters in the Forchheimer equation [9], it systematically quantified the role played by the parameters and their impact on the value of the criterion. Another study [10], uses the FLUENT software to model and perform a sensitivity analysis of the flow of a fluid in a confined aquifer. It follows from this study that the fluid flow, the porosity and the permeability of the aquifer, have a great impact on the pressure distribution in the reservoir. Reference [11] conducted a sensitivity analysis and parameter identification study for the transport of colloids in porous media. It was found that the model is very sensitive to the hydraulic parameters of the porous medium and to the transport parameters. Morris's method was improved by [12] and used to study the sensitivity of the input parameters of a porous flow model based on the Richards equation. Interesting results were obtained after cross-validation by Sobol's variance-based method. It follows from the results of this study that the size of the special discretization step is the main cause of the observed differences. Another study [13], uses sensitivity analysis to study the uncertainties of the Richards equation in a heterogeneous, unsaturated porous medium. This study uses an elementary sensitivity analysis which could be easily transformed into approximations of the functional sensitivities into limit flux sensitivities. The results of the one-dimensional numerical implementation were able to show compatibility with exact analytical solutions and with numerical disturbance calculations. The sensitivity analysis of a coupled model of heat and mass transfer in porous unsaturated media has been studied by [14]. They observe the influence of the phase conversion coefficient on the output of the model. To do this, the authors consider the observed parameter as a variable, and the derivative with respect to the variable gives the measure of the sensitivity. Thus, it follows that the phase conversion coefficient has little influence on the model. For the search for globally optimized input parameters, the uncertainty values used in the global sensitivity analysis based on the variance of the Sobol method can be subjected to an inverse method. This was used by [15], for a model of soil water infiltration, using a simple genetic algorithm to search for an optimized set of input parameters.

Using the global sensitivity analysis, based on the Sobol method, can produce a ranking of parameters that can be useful in checking over-parametrization and equifinality [16]-[18]. In the field of hydrological studies based on conceptual or uncoupled models, the use of the Sobol method is successful [16], [19]-[23]. A good quantification of the input parameters of a transport model in porous media is obtained using a global sensitivity analysis based on Sobol indices [24]. With the adoption of a surrogate model to describe the transport, the polynomial chaos expansion theory allows a reduction in the computational time needed to predict the response of the system and is favorable to a good estimation of the Sobol indices [24]-[26]. Reference [26], uses a Markov Monte Carlo chain-based sampling method to estimate the hydraulic parameters of a flow model in a porous medium. The results obtained are in agreement with the overall sensitivity analysis study that was carried out.

In this work, we are interested in the study of the sensitivity analysis of the input parameters of a multiphase flow model in porous media. This is a continuation of the work of [27], and [28] modeling multiphase compressible flows in an oil reservoir. The studied model is a complex model. Taking into account the criteria presented in [1], on the choice of the overall sensitivity analysis method, we have chosen the Sobol method.

II. Materials and Methods

A. Model Presentation

The studied model is the one defined in [27], [28] as mentioned in the introduction section. It is a model describing multi-component multiphase flows with mass transfer between the oil and gas phases, in a porous medium, the application of which was made for the Tsimiroro Madagascar oil field [27], [28], it consists of the mass conservation equations for each component in each phase (water, oil and gas):

$$\frac{\partial}{\partial t}\left(\emptyset \frac{\rho_{Ws}}{B_w} S_w\right) - \nabla\left(\frac{\rho_{Ws}}{B_w}\frac{k_{rw}}{\mu_w}\boldsymbol{K}(\nabla p_w - \gamma_w \boldsymbol{\nabla Z})\right) = \boldsymbol{q}_W \tag{1}$$

$$\frac{\partial}{\partial t}\left(\frac{\emptyset \rho_{Os}}{B_o} S_o\right) - \nabla\left(\frac{\rho_{Os}}{B_o}\frac{k_{ro}}{\mu_o}\boldsymbol{K}(\nabla p_o - \gamma_o \boldsymbol{\nabla Z})\right) = \boldsymbol{q}_O \tag{2}$$

$$\frac{\partial}{\partial t}\left[\emptyset\left(\frac{\rho_{Gs}}{B_g} S_g + \frac{R_{So}\rho_{Gs}}{B_o} S_o\right)\right] - \nabla\left(\frac{\rho_{Gs}}{B_g}\frac{k_{rg}}{\mu_g}\boldsymbol{K}\left(\nabla p_g - \gamma_g \boldsymbol{\nabla Z}\right) + \frac{R_{So}\rho_{Gs}}{B_o}\frac{k_{ro}}{\mu_o}\boldsymbol{K}(\nabla p_o - \gamma_o \boldsymbol{\nabla Z})\right) = \boldsymbol{q}_G \tag{3}$$

It is a model of nonlinear partial differential equations, of which a finite volume scheme was used for the discretization and an implicit Euler scheme was used for the temporal discretization [27], [28], which gave the discretized system constituted by the system of (4):

$$\begin{cases} R_w = \emptyset_K \rho_{Ws}\left(\frac{S_{w,K}^{n+1}}{B_w(p_K^{n+1})} - \frac{S_{w,K}^{n}}{B_w(p_K^{n})}\right)\frac{|K|}{\Delta t} - \sum_{\{\sigma\}=\bar{K}\cap\bar{L}} \tau_\sigma(\Lambda_w)_{K/L}\left(p_L^{n+1} - p_K^{n+1} - \gamma(Z_L - Z_K)\right) + Q_{lim,K}^{w} = 0 \\ R_o = \phi_K \rho_{Os}\left(\frac{S_{o,K}^{n+1}}{B_o(p_K^{n+1}, c_K^{n+1})} - \frac{S_{o,K}^{n}}{B_o(p_K^{n}, c_K^{n})}\right)\frac{|K|}{\Delta t} - \sum_{\{\sigma\}=\bar{K}\cap\bar{L}} \tau_\sigma(\Lambda_o)_{K/L}\left(p_L^{n+1} - p_K^{n+1} - \gamma(Z_L - Z_K)\right) + Q_{lim,K}^{o} = 0 \\ R_g = \emptyset_K \rho_{Gs}\left[\left(\frac{S_{g,K}^{n+1}}{B_g(p_K^{n+1})} + \frac{R_{So}(c_K^{n+1})S_{o,K}^{n+1}}{B_o(p_K^{n+1}, c_K^{n+1})}\right) - \left(\frac{S_{g,K}^{n}}{B_g(p_K^{n})} + \frac{R_{So}(c_K^{n})S_{o,K}^{n}}{B_o(p_K^{n}, c_K^{n})}\right)\right]\frac{|K|}{\Delta t} + \\ - \sum_{\{\sigma\}=\bar{K}\cap\bar{L}} \tau_\sigma(\Lambda_{os})_{K/L}\left(p_L^{n+1} - p_K^{n+1} - \gamma(Z_L - Z_K)\right) + Q_{lim,K}^{o} + \\ - \sum_{\{\sigma\}=\bar{K}\cap\bar{L}} \tau_\sigma(\Lambda_g)_{K/L}\left(p_L^{n+1} - p_K^{n+1} - \gamma(Z_L - Z_K)\right) + Q_{lim,K}^{g} = 0 \end{cases} \tag{4}$$

As we can notice, this system is nonlinear, and the authors of the works [27], [28], used a Newton Raphson algorithm for the linearized one, which gave a linear system given by (5):

$$\delta \boldsymbol{X}^{k+1} = -\boldsymbol{R}^{k} \tag{5}$$

In this equation, $\boldsymbol{J}^k$ is the Jacobian matrix, at Newton's iteration k. $\delta \boldsymbol{X}^{k+1}$ is the vector of change of unknowns such that:$\delta \boldsymbol{X}^{k+1} = \boldsymbol{X}^{k+1} - \boldsymbol{X}^{k}$.

The Jacobean matrix is given by (6):

$$\boldsymbol{J} = \begin{bmatrix} (\frac{\partial \boldsymbol{R}_1}{\partial X_1}) & \cdots & (\frac{\partial \boldsymbol{R}_1}{\partial X_N}) & (\frac{\partial \boldsymbol{R}_1}{\partial X_{well,1}}) & \cdots & (\frac{\partial \boldsymbol{R}_1}{\partial X_{well,m}}) \\ \vdots & & \vdots & \vdots & & \vdots \\ (\frac{\partial \boldsymbol{R}_N}{\partial X_1}) & \cdots & (\frac{\partial \boldsymbol{R}_N}{\partial X_N}) & (\frac{\partial \boldsymbol{R}_N}{\partial X_{well,1}}) & \cdots & (\frac{\partial \boldsymbol{R}_N}{\partial X_{well,m}}) \\ (\frac{\partial \boldsymbol{R}_{well,1}}{\partial X_1}) & \cdots & (\frac{\partial \boldsymbol{R}_{well,1}}{\partial X_N}) & (\frac{\partial \boldsymbol{R}_{well,1}}{\partial X_{well,1}}) & \cdots & (\frac{\partial \boldsymbol{R}_{well,1}}{\partial X_{well,m}}) \\ \vdots & & \vdots & \vdots & & \vdots \\ (\frac{\partial \boldsymbol{R}_{well,m}}{\partial X_1}) & \cdots & (\frac{\partial \boldsymbol{R}_{well,m}}{\partial X_N}) & (\frac{\partial \boldsymbol{R}_{well,m}}{\partial X_{well,1}}) & \cdots & (\frac{\partial \boldsymbol{R}_{well,m}}{\partial X_{well,m}}) \end{bmatrix} \tag{6}$$

The sub-matrices of the matrix (6), are given by:

$$(\frac{\partial \boldsymbol{R}_i}{\partial \boldsymbol{X}_j}) = \begin{bmatrix} \frac{\partial R_{wi}}{\partial p} & \frac{\partial R_{wi}}{\partial S_w} & \frac{\partial R_{wi}}{\partial S_g} \\ \frac{\partial R_{oi}}{\partial p} & \frac{\partial R_{oi}}{\partial S_w} & \frac{\partial R_{oi}}{\partial S_g} \\ \frac{\partial R_{gi}}{\partial p} & \frac{\partial R_{gi}}{\partial S_w} & \frac{\partial R_{gi}}{\partial S_g} \end{bmatrix} \tag{7}$$

The Matlab computer code developed by [28], will be used in this work for the study of global sensitivity analysis. Indeed, as stated in the introduction, this work is the extension of [27], [28].

B. Global Sensitivity Analysis: Method based on Analysis of Variance

Global sensitivity analysis based on variance is a method inspired by the work of Cukier [1] by Sobol who generalized it to provide a simple tool based on Monte Carlo concept implementation, capable of calculating measures of sensitivity for arbitrary groups of factors.

Consider an integrable square function f on Ω^k the unit hyper cube of dimension$k$:

$$\Omega^k = (X|0 \leq x_i \leq 1; i = 1, \dots, k) \tag{8}$$

Sobol considers an expansion of f in terms of increasing dimensions:

$$f = f_0 + \sum_i f_i + \sum_i \sum_{j>i} f_{ij} + \cdots + f_{1,2,\dots,k} \tag{9}$$

Where,

$$f_i = f_i(X_i) \quad (10)$$
$$f_{ij} = f_{ij}(X_i, X_j) \quad (11)$$

And so on,

This decomposition method is called "high dimensional model representation (HDMR)" and is not unique, which means that for a given model of f, there could be endless choices for these terms. Sobol proved that if each term in the expansion of (9) has zero mean, then all the terms of the decomposition are orthogonal. Consequently, these terms can be computed unambiguously by using the conditional expectations of the output of the model Y. These terms are computed by the relations (12) to (14):

$$f_0 = E(Y) \quad (12)$$
$$f_i = E(Y|X_i) - E(Y) \quad (13)$$
$$f_{ij} = E\left(Y\middle|X_i, X_j\right) - f_i - f_j - E(Y) \quad (14)$$

The (15) describes the first order sensitivity index:

$$S_i = \frac{V[E(Y|X_i)]}{V(Y)} \quad (15)$$

It represents the contribution to the main effect of each input parameter to the output variance. By square integrating the terms of the decomposition (9) on Ω^k, we can obtain the decomposition of the variance called ANOVA-HDMR:

$$V(Y) = \sum_i V_i + \sum_i \sum_{j>i} V_{ij} + \cdots + V_{1,2,\ldots,k} \quad (16)$$

Where $V_i = V(f_i)$; $V_{ij} = V\left(f_{ij}\right)$, so on, are the associated variances. V_{ij} is the joint effect between X_i *and* X_j minus the first order effects for the same factors. By dividing (16) member by member by $V(Y)$ we obtain:

$$\sum_i S_i + \sum_i \sum_{j>i} S_{ij} + \sum_i \sum_{j>i} \sum_{l>j} S_{ijl} + \cdots + S_{1,2,3,\ldots,k} = 1 \quad (17)$$

The number of terms in (17) increases exponentially with the number of input factors. The S_{ij}, S_{ijl},…, are the effects of second order, third order and so on.

From (17), we can define the total effect of each parameter, which is the first order effect plus all higher order effects. That is, for the parameter $X_i$whose first order effect index is S_i, its total effect will be defined by:

$$S_{Ti} = S_i + \sum_i \sum_{j>i} S_{ij} + \sum_i \sum_{j>i} \sum_{l>j} S_{ijl} + \cdots + S_{1,2,3,\ldots,k} \quad (18)$$

This index represents the contribution of total to the production of variation due to the factor X_i. Decomposing the variance $V(Y)$ in terms of the main and residual effect by conditioning all the factors except one ($X_{\sim i}$) we can calculate the total index:

$$S_{Ti} = \frac{E[V(Y|X_{\sim i})]}{V(Y)} = 1 - \frac{V[E(Y|X_{\sim i})]}{V(Y)} \quad (19)$$

C. *Sensitivity Index Calculation Algorithm*

The algorithm proposed in this work is inspired by the work of [1] and [31], and allows easy calculation of the parametric sensitivity indices of the model using analysis based on variance. The algorithm is easy to implement in Matlab. In this algorithm, k is the number of input parameters of the model.

```
// N and k are the base sample and the number of input factors
//   Given the maximum value and the minimum value for each factor F, a matrix of two columns
//and k rows: each rows of F contains the minimum value and the maximum value of  ith factor,
//where i varies from 1 to k.
k ← size(F,1) ;
// generate two (N,k)  A and B matrix respectively  of random numbers
 A ← [F(1,1) + (F(1,2) − F(1,1)).∗ rand(N,1), F(2,1) + (F(2,2) − F(2,1)).∗ rand(N,1), … ,
                         F(k,1) + (F(k,2) − F(k,1)).∗ rand(N,1)] ;
// we use some equationwe used to calculate the A matrix but the two matrices are different
//because of the random values generated in rand
 B ← [F(1,1) + (F(1,2) − F(1,1)).∗ rand(N,1), F(2,1) + (F(2,2) − F(2,1)).∗ rand(N,1), … ,
                         F(k,1) + (F(k,2) − F(k,1)).∗ rand(N,1)] ;
// define a matrix C_i formed by all columns of  B except the ith column which is taken from A and
compute the model output for all the imput values of the sample matrices A, B and C_i
for  i  ← 1 to N
      xa ← A(i,:);// vector contains all input values for each i
      // compute y_A = f(xa)
      y_A(i,:) ← Model(xa);
     // compute y_B = f(xb)
// compute the matrix C_i and  y_{C_i} = f(xci)
     for j ← 1 to k
          xb ← [B(i,1 to j − 1), A(i,j), B(i,j + 1 to N)]; y_B(i,:,j) ← Model(xb);
          xci ← [A(i,1 to j − 1), B(i,j), A(i,j + 1 to N)]; y_ci(i,:,j) ← Model(xci);
     end
end
// Compute of the variances
n ← size(y_A,2); f_0 ← zeros(1,n); V ← zeros(1,n)  ;
for i ← 1 to N
      f_0 ← f_0 + y_A(i,:)/N;
      V ← V + y_A(i,:).^2/N;
end
V ← V − f_0^2 ;
// compute  the partial variance for each parameter  and the total partial variance
V_j ← ones(k,1) ∗ V;  V_tj ← zeros(k,n) ;
for i ← 1 to N
     for j ← 1to k
           V_j(j,:) ← V_j(j,:) − (y_A(i,:) −y_B (i,:,j)).^2/(2*N);
           V_tj(j,:) ← V_tj(j,:) + (y_A(i,:) −y_ci (i,:,j)).^2/(2*N);
     end
end
// compute the sensitivity index
S_i ← V_j/ones(k,1)*V; // first order
ST_i ← V_tj/ones(k,1)*V; // total effect
// sort sensitivity ranking
[Si,r] = sort(S_i) ; [STi,r] = sort(S_Ti)
```

Fig. 1. Algorithm for calculating sensitivity indices.

III. Results and Discussions

Fig. 2 and 3, show the influence of the input parameters of the multi-component multiphase flow model on the pressure and oil saturation output parameters. It can be seen from these results that the most influential parameters on the pressure are the number of discretization according to each direction of the field, the injection pressure, the dimensions of the reservoir, and the initial pressure of the reservoir, the porosity, and the permeability of the porous medium. On the other hand, we can see that the imposed production pressure and the simulation time have almost no effect on the output pressure of the model. This observation agrees with the obtained results in [28], on the convergence of the model. Note that the comparison of the first order sensitivity indices with the total indices, shows that almost every input parameter of the model can interact with other input parameters of the model. Indeed, the difference $S_{Ti} - S_i$is a measure of how much the parameter $X_i$is involved in interactions with any other parameter [1]. This on the one hand explains the complexity of the model and therefore on the other hand imposes an optimization of the model for the right choice of simulation parameters.

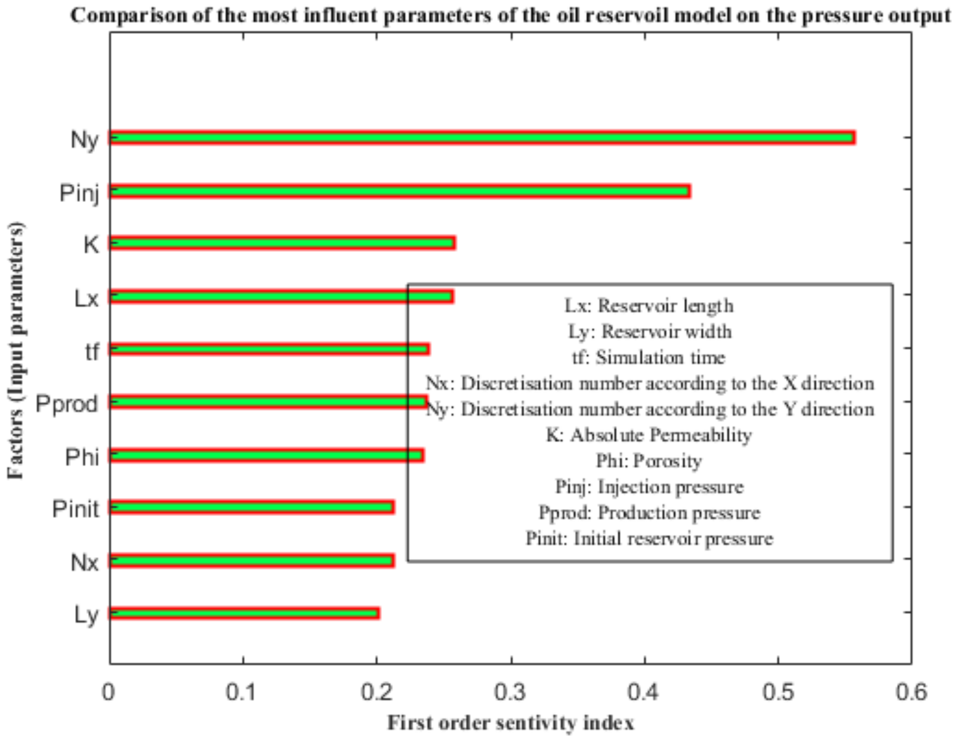

Fig. 2. First Order sensitivity index effect for the pressure output parameter.

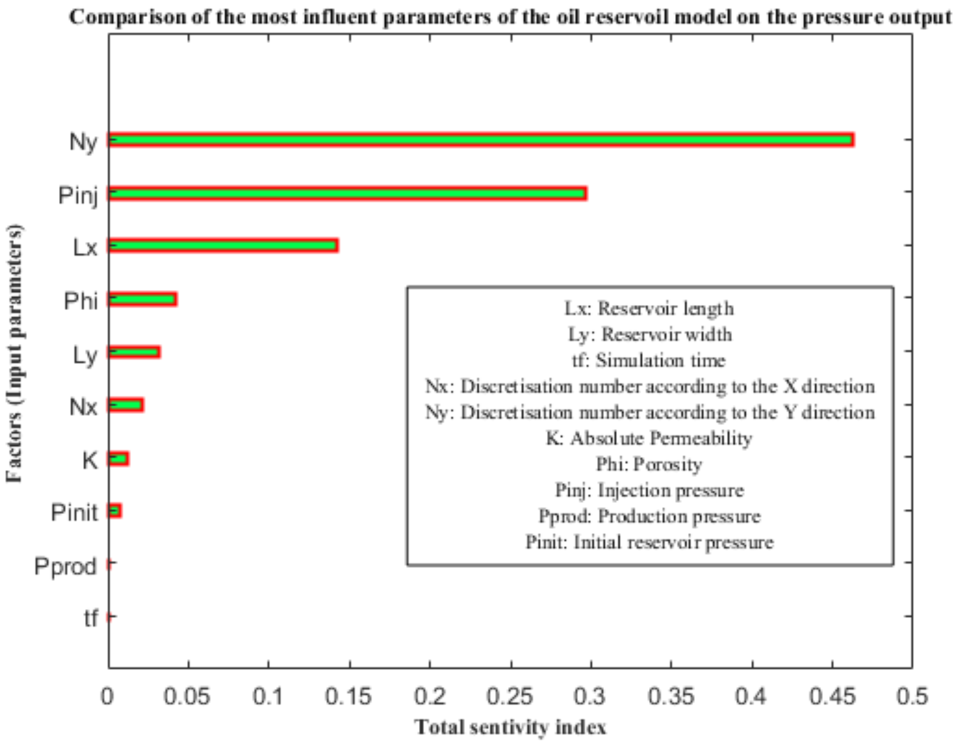

Fig. 3. Total sensitivity for the pressure output parameter.

Fig. 4 and 5 make it possible to carry out the same analysis as that made for Fig. 2 and 3. However, the order of influence of the porous medium absolute permeability and porosity parameters is much greater on the oil saturation output parameter than on the pressure output parameter. In fact, the porosity and the absolute permeability characterize the porous medium and respectively describe the percentage of oil contained in the porous medium relative to the total volume of the available porous medium and the capacity of the porous medium to let the fluid flow in; hence the good choice of these parameters which are often difficult to measure and / or estimate.

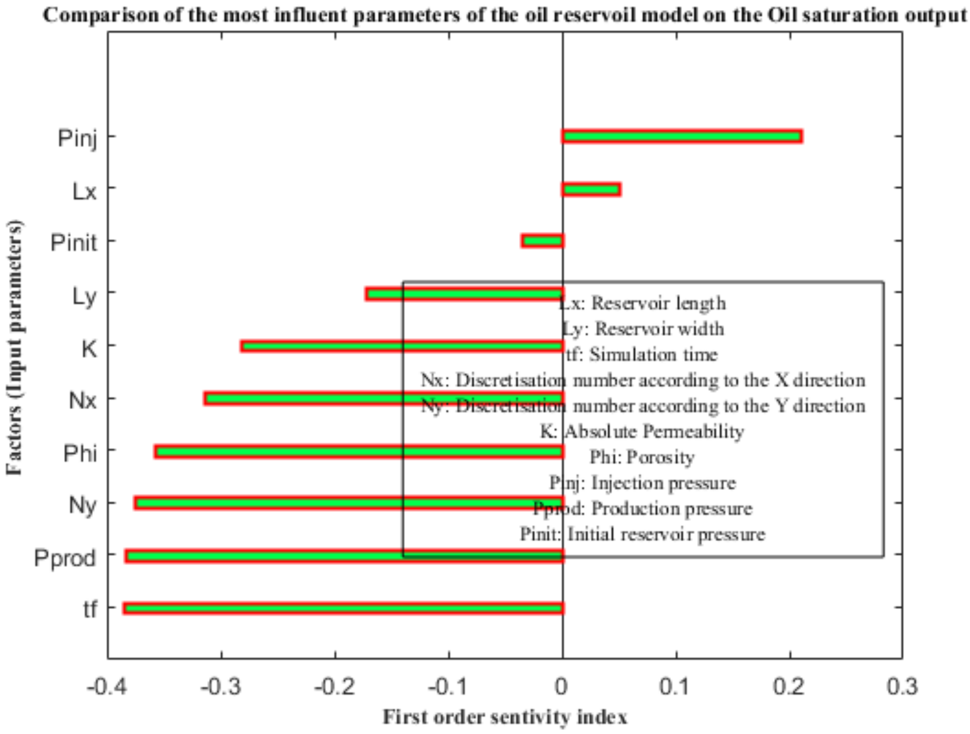

Fig 4. First order index for oil saturation output parameter.

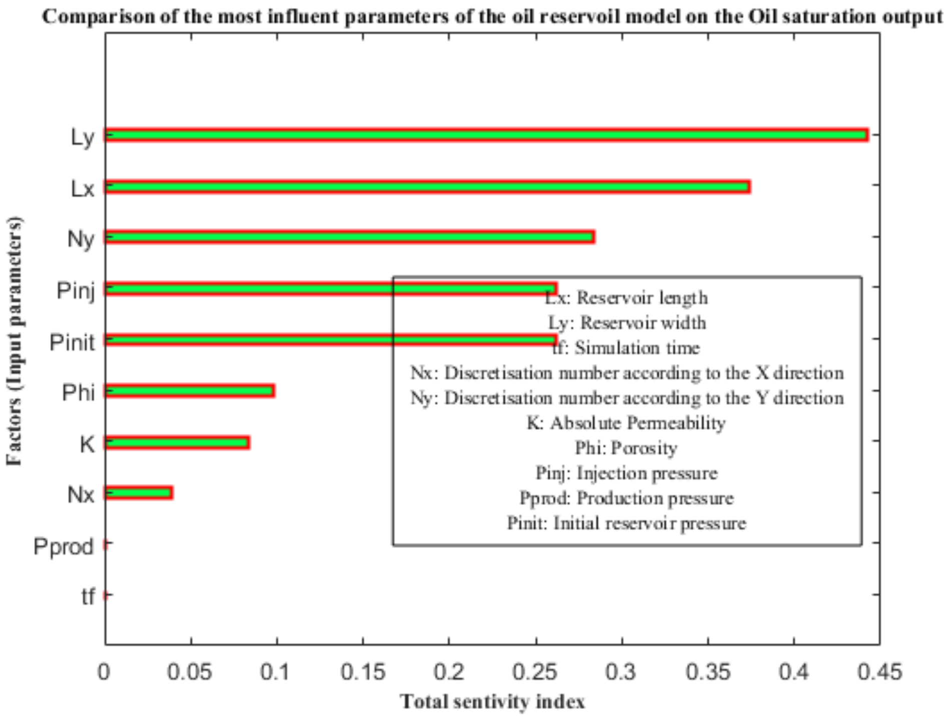

Fig. 5. Total sensitivity index for oil saturation output parameter.

IV. Conclusion

The work presented in this article consists of parametric sensitivity analysis of a multi-component multiphase flow model in a porous geological medium. It was found that the input parameters of the model can be in interaction with each other, but especially that most of them have a lot of influence on the pressure and the oil saturation which are the observed output parameters. This means that the input parameters will have a big impact on the production of the reservoir. The analysis shows that particularly the dimensions of the reservoir and the injection pressure have a great influence on the two observed output parameters, while the initial pressure of the reservoir has a great influence on oil saturation than on the pressure which both are output parameters of the model. The porosity and the permeability of the porous medium have a lot of influence on the oil saturation while for the outlet pressure, it is the porosity which is much more influential. These two parameters are stochastic parameters, of the porous medium, that are difficult to measure or estimate with certainty.

For a better optimization of the production of the reservoir, this study allows us to draw the following conclusions:

- The choice of reservoir dimensions and the location of the wells must be crucial,
- Estimate the initial pressure of the reservoir and the physical parameters such as porosity and permeability,
- Adjust the injection pressure carefully for maximum production,
- As for the discretization, the choice of the discretization step is of capital importance despite the cost in computing time.

To better understand the functioning of the model and improve the optimization, a lot of work remains to be done, including the sensitivity analysis of the relative permeability models to see which model will have a better impact, but also the study of capillary pressure models. It would also be interesting to examine the case where the middle fractures are taken into account in the model.

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CONFLICT OF INTEREST

Authors declare that they do not have any conflict of interest.

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