The Study of Hemodynamic in Stenosed Human Arteries with Sodium Alginate Nanoparticles as A Drug Carrier

Nitin Sadashiv Bodke and Ignatius Fernandes

***Abstract* -** **Blood flow in stenosed human arteries in the presence of sodium alginate nanoparticles is simulated. Effect of sodium alginate (SA) nanoparticles on velocity, flow rate and resistive impedance to blood flow in stenosed human arteries is studied. The equations governing blood flow are discretized using finite differences. The MATLAB software is used to simulate the discretized equations. Presence of nanoparticles is observed to influence velocity, flow rate and impedance to blood flow. Resistance to flow is observed to be less in the presence of nanoparticles. This nanoparticle drug delivery may be useful for patients having cardiovascular diseases.**

Keywords: *Nanoparticles, stenosed artery, simulation.*

I. INTRODUCTION

Atherosclerosis is the disease which is most commonly observed in cardiovascular vessels of human beings. This disease is caused due to accumulation of fatty material inside the artery. This deposition causes depletion of space of the lumen of the artery which leads to cardiovascular disorders. Therefore, it is important to increase blood flow in the diseases part of the artery.

Transport of nanoparticles under blood flow using an agent-based approach was studied by [1]. Blood-mediated nanoparticle delivery is implemented in their paper. Reference [2] presented a theoretical analysis of metallic nanoparticles on blood flow through tapered elastic arteries with overlapping stenosis. They computed the expression for impedance resistance to flow and wall shear stress. Reference [3] studied transport of nanoparticles in blood vessels. They described blood to be Casson fluid. Relation of rheology of blood and the permeability of the vessels with delivery of nanoparticles is observed. Reference [4] studied the effect of copper (Cu) and silver (Ag) nanoparticles on blood flow through a curved stenosed channel with aneurysm. They have solved the nano-particles momentum and thermal energy equations numerically using explicit finite difference schemes.

Reference [5] investigated the effect of adding nanoparticles to the blood flow in presence of magnetic field in a porous blood arterial. In their paper; the effect of physical parameters Brownian motion, thermophoresis and pressure gradient on velocity profile and temperature is studied. Reference [6] studied the shape effect of Cu-nanoparticles in unsteady flow through curved arteries with catheterized stenosis. Effects of nanoparticles such as Fe_3O_4, TiO_2, and Cu on blood flow inside a stenosed artery was presented by [7]. They modelled blood as non-Newtonian Bingham plastic fluid subject to periodic body acceleration. Reference [8] investigated the interaction of gold nanoparticles in blood and monitored their influence on blood coagulation.

Comparative study of behaviour of hemodynamic properties in stenosed arteries in the presence of sodium alginate (SA) nanoparticles and in the absence of nanoparticles is not yet studied. Therefore we have taken up this study. This study may be useful in the development of therapeutics and diagnostics.

II. GOVERNING EQUATIONS

The segment of stenosed artery of length L is considered as a cylindrical tube containing Newtonian fluid representing the flowing blood. The geometry of the stenosis is presented in Fig. 1. The flow is assumed to be laminar, unsteady, two dimensional and axisymmetric. The mathematical model can be expressed by conservation of mass, momentum and temperature from [2].

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\rho_{nf}\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial r} + \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u^2}{r^2} + \frac{\partial^2 u}{\partial z^2}\right) \tag{2}$$

$$\rho_{nf}\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu_{nf}\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right) +$$
$$g(\rho\gamma)_{nf}(T - T_1) \tag{3}$$

$$\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z}\right) = \left(\frac{k_{nf}}{(\rho C_p)_{nf}}\right)\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right) + \frac{Q_0}{(\rho C_p)_{nf}} \tag{4}$$

Where u and w are radial and axial velocities respectively. $\mu_{nf}, \rho_{nf}, k_{nf}$ and γ_f are viscosity, density, thermal conductivity, coefficient of thermal expansion of nanofluids. T is temperature of fluid, Q_0 is constant of heat absorption or heat generation.

The dynamic viscosity μ_{nf} of the nanofluid is given as [9]:

$$\mu_{nf} = \frac{\mu_f}{(1 - \emptyset)^{2.5}}$$

TABLE I: Physical Values of Blood and Nanoparticle are given by [4] and [10]

Parameters	Blood	Sodium Alginate
C_p(J/Kg.K)	3594	4175
ρ (Kg/m^3)	1063	989
γ (1/K)	0.18	0.16
K (W/m.K)	0.492	0.6376

III. The Geometry of Stenosis

The stenosis geometry is time dependent. Multiple stenosis regions are overlapped. It is described from [11] by

$$R(z,t) = a\left\{1 - \frac{\tau_m}{5005\, a\, l_0^6}\left[\frac{668662}{9}(z-d)\, l_0^5 - 370281(z-d)^2\, l_0^4 + 743344(z-d)^3\, l_0^3 - \right.\right.$$
$$698476(z-d)^4\, l_0^2 + 307584(z-d)^5 l_0 - 51264(z-d)^6\left]\right\} a_1(t)$$
$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad \ldots\ldots d < z < d + 2l_0$$
$$= aa_1(t) \qquad\qquad\qquad\qquad \ldots\ldots \text{Otherwise}$$

Where $a_1(t) = 1 - \cos\cos(\omega t - 1)\,\beta e^{\beta\omega t}$ in which ω is angular frequency and β is the constant Parameter.
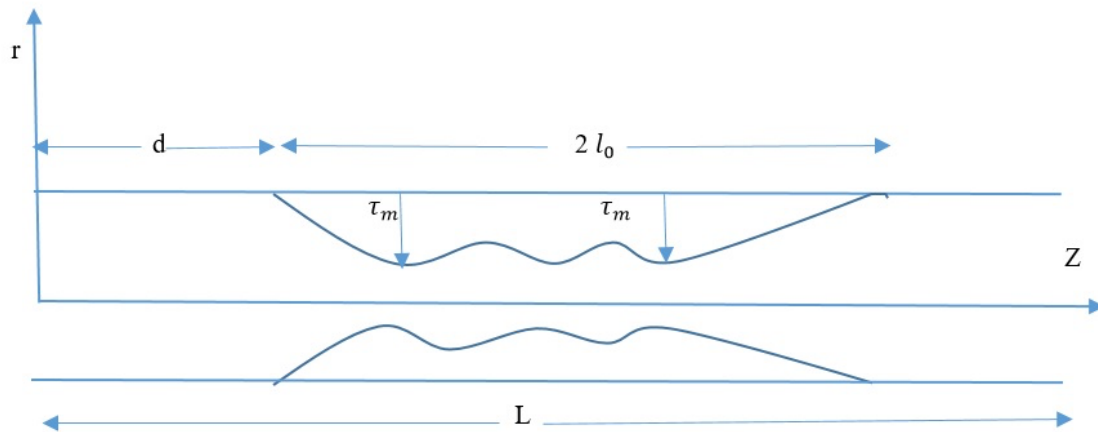

Fig. 1. The Stenosis Geometry in an Artery.

IV. Boundary Conditions

The velocities at the inlet and outlet of an arterial segment of finite length are taken as [2] and [12]

$$u(r,z,t) = 0 \text{ and, } w(r,z,t) = \frac{5}{3}\left(1 - \left(\frac{r}{R(z,t)}\right)^3\right) \text{ at } z = 0 \tag{5}$$

$$\frac{\partial w(r,z,t)}{\partial z} = 0 = \frac{\partial u(r,z,t)}{\partial z} \text{ at } z = L \tag{6}$$

It is assumed that initially radial and axial velocity both are zero. That is when system is at rest there is no flow through artery

i.e. $u(r, z, 0) = 0, w(r, z, 0) = 0, T(r, z, 0) = 0$ (7)

Axially, there is no radial flow, therefore the radial velocity is zero, the axial velocity gradient of the blood and temperature gradient may be assumed to be equal to zero. This may be stated as

$$\frac{\partial w}{\partial r} = 0, \; u(r, z, t) = 0, \; \frac{\partial T}{\partial r} = 0 \text{ on } r = 0 \tag{8}$$

On the artery wall the axial velocity is zero due to no slip condition, temperature of the fluid is zero and radial velocity is rate of change in shape of the stenosis, which can be written as

$$w(r, z, t) = 0, \; u(r, z, t) = \frac{\partial R}{\partial t}, \; T(r, z, t) = 0 \text{ on } r = R(z, t) \tag{9}$$

V. Numerical Method and Implementation

Equations (3), (4) along with boundary conditions (5)-(9) take following form after introducing radial coordinate transformation, $x = \frac{r}{R(z,t)}$

$$
\frac{\partial w}{\partial t} = \frac{1}{R}\left[x\left(w\frac{\partial R}{\partial z} + \frac{\partial R}{\partial t}\right) - u\right]\frac{\partial w}{\partial x} - w\frac{\partial w}{\partial z} + \frac{\mu_{nf}}{\rho_{nf}R^2}\left(\frac{\partial^2 w}{\partial x^2} + \frac{1}{x}\frac{\partial w}{\partial x}\right) - \frac{1}{\rho_{nf}}\frac{\partial p}{\partial z}
$$
$$
+ \; g(\gamma)_{nf}(T - T_1) \tag{10}
$$

$$
\frac{\partial T}{\partial t} = \frac{1}{R}\left[x\left(T\frac{\partial R}{\partial z} + \frac{\partial R}{\partial t}\right) - u\right]\frac{\partial T}{\partial x} - T\frac{\partial T}{\partial z} + \left(\frac{k_{nf}}{(\rho C_p)_{nf}}\right)\frac{1}{R^2}\left(\frac{\partial^2 T}{\partial x^2} + \frac{1}{x}\frac{\partial T}{\partial x}\right) + \frac{Q_0}{(\rho C_p)_{nf}} \tag{11}
$$

$$u(x, z, t) = 0 \text{ and }, w(x, z, t) = \frac{5}{3}(1 - x^3) \text{ at } z = 0 \tag{12}$$

$$\frac{\partial w(x,z,t)}{\partial z} = 0 = \frac{\partial u(x,z,t)}{\partial z} \text{ at } z = L \tag{13}$$

$$u(x, z, 0) = 0, w(x, z, 0) = 0 \qquad , T(x, z, 0) = 0 \tag{14}$$

$$\frac{\partial w}{\partial x} = 0, \; u(x, z, t) = 0, \; \frac{\partial T}{\partial x} = 0 \text{ on } x = 0 \tag{15}$$

$$w(x, z, t) = 0, \; u(x, z, t) = \frac{\partial R}{\partial t}, \; T(x, z, t) = 0 \text{ on } x = 1 \tag{16}$$

Solving (10) and (11) using finite difference approximations in which central differences have been used.

$$\frac{\partial w}{\partial x} = \frac{w_{i,j+1}^k - w_{i,j-1}^k}{2\Delta x}$$
$$\frac{\partial w}{\partial z} = \frac{w_{i+1,j}^k - w_{i-1,j}^k}{2\Delta z}$$
$$\frac{\partial w}{\partial t} = \frac{w_{i,j}^{k+1} - w_{i,j}^k}{\Delta t}$$
$$\frac{\partial^2 w}{\partial x^2} = \frac{w_{i,j+1}^k - 2w_{i,j}^k + w_{i,j-1}^k}{(\Delta x)^2}$$

Where $x_j = (j - 1)\Delta x$, $z_i = (i - 1)\Delta z$ and $t_k = (k - 1)\Delta t$.

$\Delta x, \Delta z$, are increments in radial, axial directions respectively.

$$
w_{i,j}^{k+1} = w_{i,j}^k + \Delta t\left\{\frac{-1}{\rho_{nf}}\left(\frac{\partial p}{\partial z}\right)^{k+1} - w_{i,j}^k\left(\frac{w_{i+1,j}^k - w_{i-1,j}^k}{2\Delta z}\right) + \left(\frac{x_j}{R_i^k}w_{i,j}^k\left(\frac{\partial R}{\partial z}\right)_i^k + \frac{x_j}{R_i^k}\left(\frac{\partial R}{\partial t}\right)_i^k - \right.\right.
$$
$$
\left.\left. \frac{u_{i,j}^k}{R_i^k}\right)\left(\frac{w_{i,j+1}^k - w_{i,j-1}^k}{2\Delta x}\right) + \frac{\mu_{nf}}{\rho_{nf}(R_i^k)^2}\left[\frac{w_{i,j+1}^k - 2w_{i,j}^k + w_{i,j-1}^k}{(\Delta x)^2} + \frac{1}{x_j}\left(\frac{w_{i,j+1}^k - w_{i,j-1}^k}{2\Delta x}\right)\right] + g(\gamma)_{nf}(T - T_1)\right\} \tag{17}
$$

$$
T_{i,j}^{k+1} = T_{i,j}^k + \Delta t\left\{T_{i,j}^k\left(\frac{T_{i+1,j}^k - T_{i-1,j}^k}{2\Delta z}\right) + \left(\frac{x_j}{R_i^k}T_{i,j}^k\left(\frac{\partial R}{\partial z}\right)_i^k + \frac{x_j}{R_i^k}\left(\frac{\partial R}{\partial t}\right)_i^k - \frac{u_{i,j}^k}{R_i^k}\right)\left(\frac{T_{i,j+1}^k - T_{i,j-1}^k}{2\Delta x}\right) + \right.
$$
$$
\left.\left(\frac{k_{nf}}{(\rho C_p)_{nf}}\right)\frac{1}{R_i^k}\left[\frac{T_{i,j+1}^k - 2T_{i,j}^k + T_{i,j-1}^k}{(\Delta x)^2} + \frac{1}{x_j}\left(\frac{T_{i,j+1}^k - T_{i,j-1}^k}{2\Delta x}\right)\right] + \frac{Q_0}{(\rho C_p)_{nf}}\right\} \tag{18}
$$

MATLAB simulation is used to solve (17) and (18) with boundary conditions (12)-(16). Flow rate can be computed using

$$Q_i^k = 2\pi(R_i^k)^2\int_0^1 x_j w_{i,j}^k \, dx_j \tag{19}$$

Resistance to flow (Resistive Impedance) is calculated using

$$X_i^k = \frac{\left| L\left(\frac{\partial p}{\partial z}\right)^k \right|}{Q_i^k} \tag{20}$$

VI. RESULT

Following numerical data is used for calculation.

d = 0.005 m, $l_0 = 0.01\ m$, $L = 0.03\ m$, $a = 0.0008\ m$, $f_p = 1.2\ Hz$, $\mu = 0.035\ P$, $\tau_m = 0.2a$, $A_0 = 100\ Kg\ m^{-2}s^{-2}$, $A_1 = 0.2A_0$, $\beta = 0.1$

Fig. 2 and 3 represent axial variation of axial velocity without nanoparticles and at 8% nanoparticles concentration. Axial velocity follows the shape of the stenosis. It is more in the presence of nanoparticles.

Fig. 4 and 5 represent distribution of flow rate along the length of an artery without nanoparticles and at 8% nanoparticles concentration. Comparison of Fig. 4 and 5 show that flow rate increases in the presence of nanoparticles.
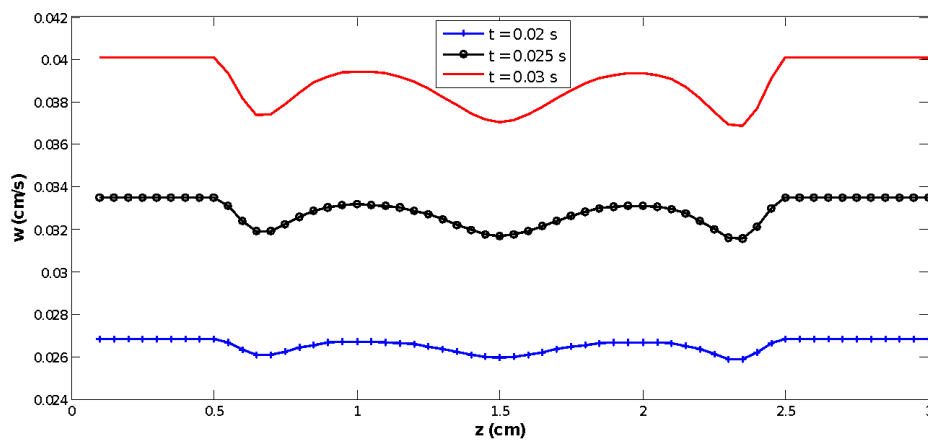

Fig. 2. Axial Variation of Axial Velocity without Nanoparticles.
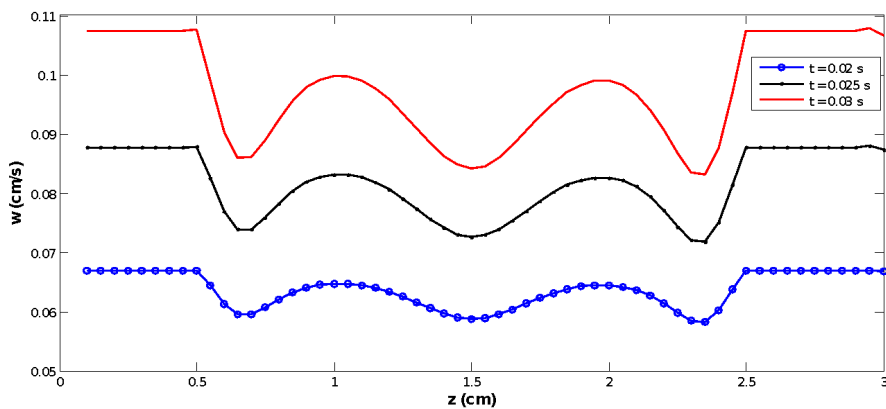

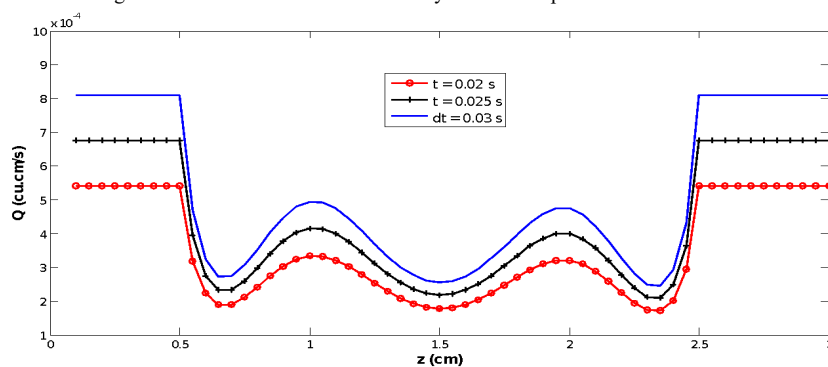Fig. 3. Axial Variation of Axial Velocity at 8% Nanoparticle Concentration.

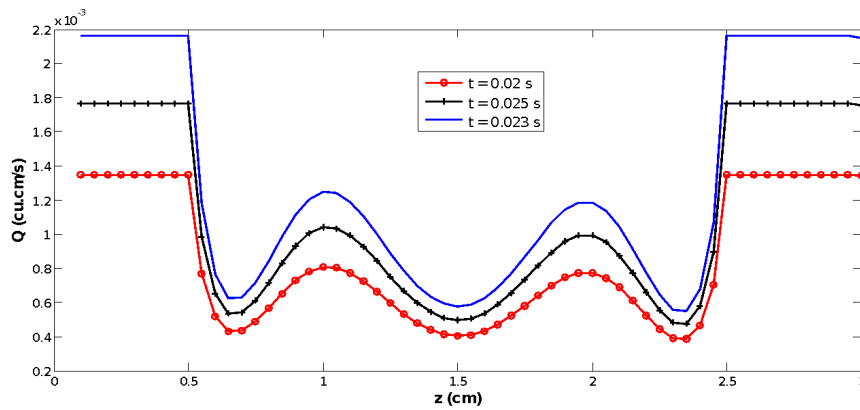Fig. 4. Flow Rate Along the Length of the Artery without Nanoparticles.

Fig. 5. Flow Rate Along the Length of the Artery at 8% Nanoparticle Concentration.

Fig. 6 and 7 represent resistive impedance in the considered arterial segment without nanoparticles and at 8% nanoparticles concentration. It is observed that resistance to the flow is less in the presence of nanoparticles.
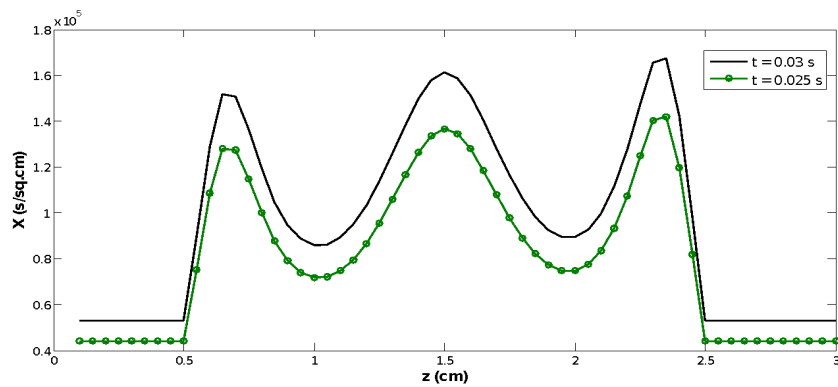

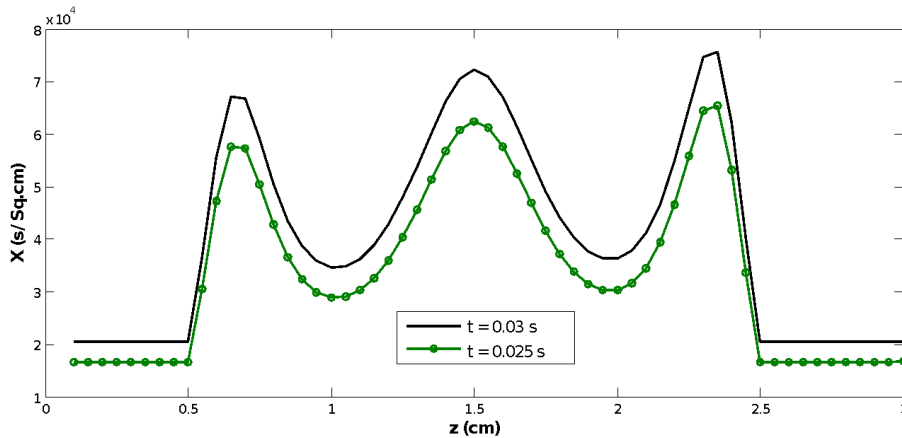Fig. 6. Resistive Impedance Along the Length of the Artery without Nanoparticles.

Fig. 7. Resistive Impedance along the Length of the Artery at 8 % nanofluid concentration.

VII. Conclusion

Nanoparticle drug delivery is observed to increase blood velocity, flow rate in diseased part of an artery. Also the resistance to blood flow in the diseased part of the artery is less in the presence of nanoparticles. This can be used to increase blood flow in the diseased part of the artery in cardiovascular patients.

Conflict of Interest

Authors declare that they do not have any conflict of interest.

REFERENCES

[1] Fullstone G, Wood J, Holcombe M, Battaglia G. Modelling the transport of nanoparticles under blood flow using an agent-based approach. *Sci. Rep.* 2015.

[2] Nadeem S, Ijaz S. Theoretical analysis of metallic nanoparticles on blood flow through tapered elastic artery with overlapping stenosis. *Phys. lett. A*. 2015; 379: 542-554.

[3] Gentile F, Ferrari M, Decuzzi P. The transport of nanoparticles in blood vessels: The effect of vessel permeability and blood rheology. *Ann Biomed Eng*. 2008; 36(2): 254-261

[4] Zaman A, Ali N, Ali I. Effects of nanoparticles (Cu (Copper), Silver (Ag)) and slip on unsteady blood flow through a curved stenosed channel with aneurysm. *Therm. Sci. Eng. Prog.* 2018; 5: 482-491.

[5] Ardahaie SS, Amiri AJ, Amouei A, Hosseinzadeh K, Ganji DD, Investigating the effect of adding nanoparticles to the blood flow in presence of magnetic field in a porous blood arterial. *Inform. Med. Unlocked*. 2018; 10: 71-81.

[6] Ashfaq A, Sohail N. Shape effect of Cu-nanoparticles in unsteady flow through curved artery with catheterized stenosis. *Results Phys.* 2017; 7: 677-689.

[7] Dzuliana FJ, Rozaini R, Mohammed A, Norziha, Che-Him, Suliadi S, et al. Unsteady blood flow with nanoparticles through stenosed arteries in the presence of periodic body acceleration. *J. Phys. Conf. Ser.* 2018; 995.

[8] Ajdari N, Vyas C, Bogan SL, Lwaleed BA, Cousins BG. Gold nanoparticle interactions in human blood: a model evaluation. *Nanomedicine: Nanotechnology, Biology and Medicine*. 2017; 13: 1531-1542.

[9] Hatami M, Nouri R, Ganji DD. Forced convection analysis for MHD Al2O3–water nanofluid flow over a horizontal plate. *J. Mol. Liq.* 2013; 187: 294–301.

[10] Hatami M, Ganji DD. Natural convection of sodium alginate (SA) non-Newtonian nanofluid flow between two vertical flat plates by analytical and numerical methods. *Case Stud. Therm. Eng.* 2014; 2: 14-22.

[11] Chakravarty S, Sannigrahi A. A nonlinear Mathematical model of blood flow in a constricted artery experiencing body acceleration. *Mathematical and Computer Modelling*. 1999; 29: 9-25.

[12] Sarifuddin, Chakravarty S, Mandal PK, Effect of heat and mass transfer on non-Newtonian flow – Links to atherosclerosis. *Int. J. Heat Mass Transf.* 2009; 52: 5719–5730.